15) Perturbed M sets and the Juliabrot

(The Article15-module is included in my ik-module. This sub-module is only a temporary module solely written for this article. The module draws the c-plane of the formula z -> z^2 + pz + c. Any initiation of "z" can be done under "starting point". Under "z-coefficient" any complex number can be set to "p". The default is set to p = 1 + 0i.)

Part I (This part is a fiction): Hurrah! I've created a new formula! When I got tired of the standard Mandelbrot set I, in a creative moment, got a flash idea according to the following: Why not, instead of drawing the parameter-plane (c-plane) in the standard Mandelbrot-formula z -> z^2 + c, insert an extra "z".

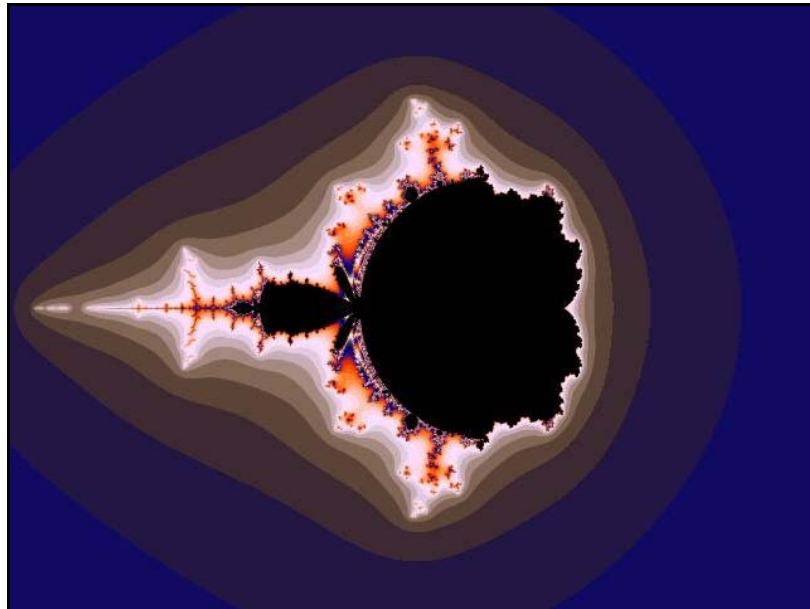**Fig 1. z^2 + z + c.**

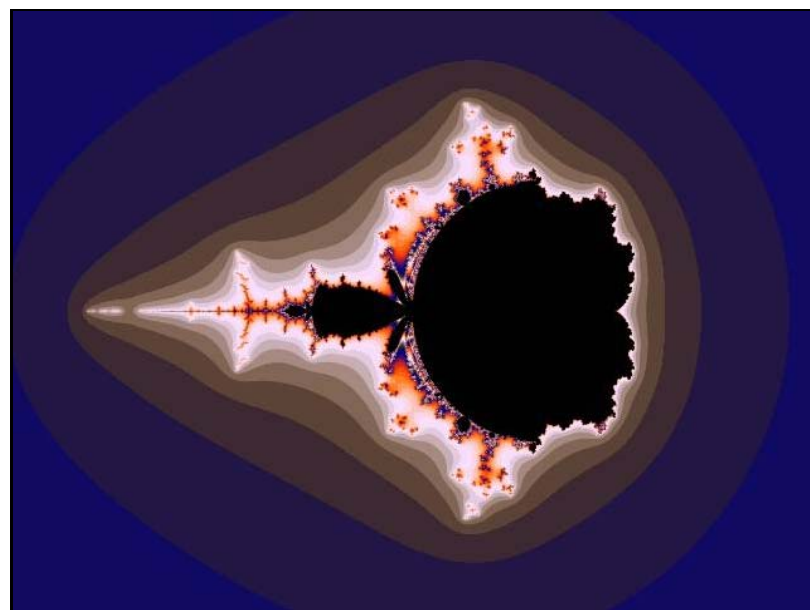**Fig 2. Perturbed.**

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That is to draw the c-plane when iterating z -> z^2 + z + c. Doing so I achieved figure 1, z^2 + z + c. However tell me that happiness that lasts forever (Swedish tale). One day my master told me. - Look here! In the standard Mandelbrot formula (z -> z^2 + c), I've changed the initiation of "z" from z = 0 to z = 0.5, and then I receive the same image as you (see figure 2, "Perturbed"). The only difference is the location along the real axis. In the formula you invented (z -> z^2 + z + c) you still use z = 0 as first input, but for this formula z = 0 is not the critical point for your genius formula. - How do I then obtain the critical point? I asked. - You put the derivative to zero. In this case we obtain:

$$p(z) = z^2 + z + c$$
$$p'(z) = 2z + 1 = 0$$
$$2z = -1$$
$$z = -1/2$$

That means that if you use z = -1/2 as input, you obtain the ordinary Mandelbrot set, the only difference being the location in the plotted plane (see
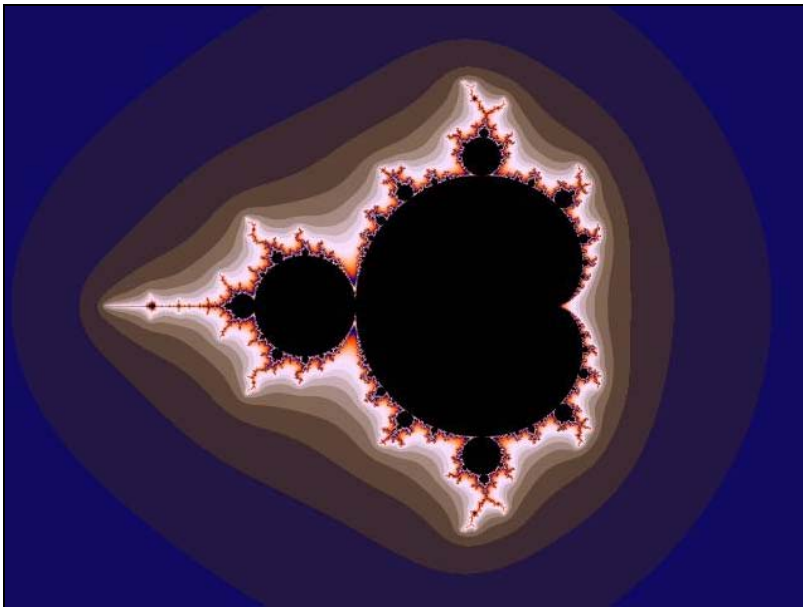
Fig 3. z^2 + z + c critical.
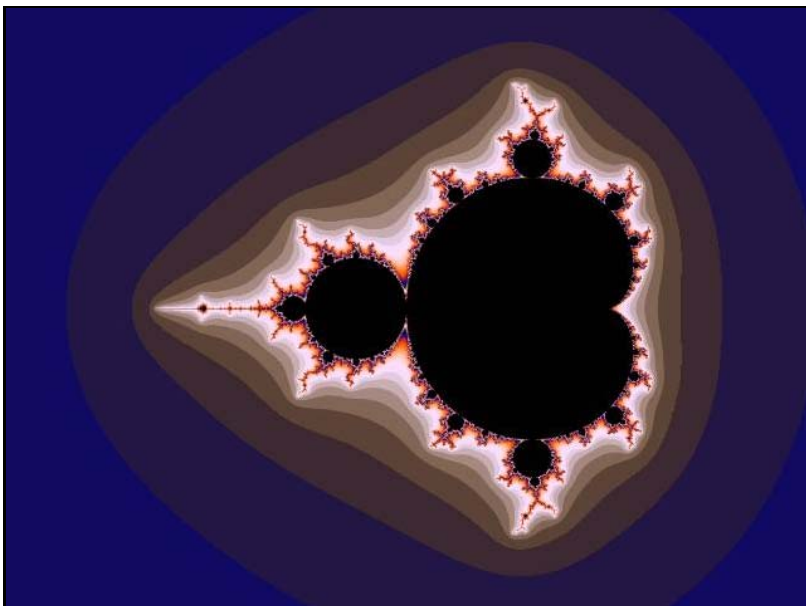
Fig 4. Standard Mandelbrot.

the figure 3, "z^2 + z + c critical").

Part II: According to the great mathematicians, all polynomials of degree two, P(Z) = AZ^2 + BZ + C, can be represented in a form with one parameter. The smartest way to display such a quadratic polynomials is in the form p(z) = z^2 + c which cause the critical point to be z = 0.

Part III: The sets received on parameter-planes initiating the variable to a non-critical number are called *perturbed sets*. If we discuss the standard iteration-formula z -> z^2 + c we can put all the 4 axis' z_{real}, z_{imag}, c_{real}, and c_{imag} together in a four-dimensional coordinate system. This builds up a four dimensional hyper space which can be studied in three- or two-dimensional slices. This can be done by fixing one or two of the axis' letting the computer plot the rest. In the case of perturbed sets we plot the plane (c_{real}, c_{imag}) while z_{real} and z_{imag} are fixed, These last axis' pointing into the third and fourth dimensions. The special case when z_{real} = 0 and z_{imag} = 0 produce the standard M set. The perturbed sets can be considered as other slices of (c_{real}, c_{imag}) when gliding along the axis z_{real} and z_{imag}.

On the other hand plotting the plane (z_{real}, z_{imag}) gliding along the axis c_{real} and c_{imag} produce all the quadratic Julia sets. In other words, the Mandelbrot set, the Perturbed M sets, and all the connected Julia sets can be considered as two dimensional slices of a four dimensional monster called *Juliabrot*.

Because a four dimensional system have six two-dimensional perpendicular systems of slices, we, in our case besides the two systems:

c_{real}, c_{imag},
z_{real}, z_{imag}
also have the systems:

z_{real}, c_{real},
z_{real}, c_{imag},
z_{imag}, c_{real},
z_{imag}, c_{imag}

to investigate. Nice applications for studying Juliabrot can be found both in the sp- and spr- module to Ultra Fractal of Stig Pettersson. All variations of 2D- and 3D-slices can be investigated with the sub-modules in both modules (In the spr-module the Juliabrot are to be found under "Raytracing").

In the next article we leave the quadratic polynomials and take the next natural step, the step to third degree or cubic polynomials. The full analogy to the Mandelbrot set for "cubics" is also a four dimensional fractal monster.

As this is a chaotic series we probably will return to the Mandelbrot set some day.

Regards
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