16) Introduction to Cubics

With this article we take the step from iteration of quadratic polynomials to iteration of cubic polynomials. As any quadratic polynomial,

$P(Z) = AZ^2 + BZ + C$,

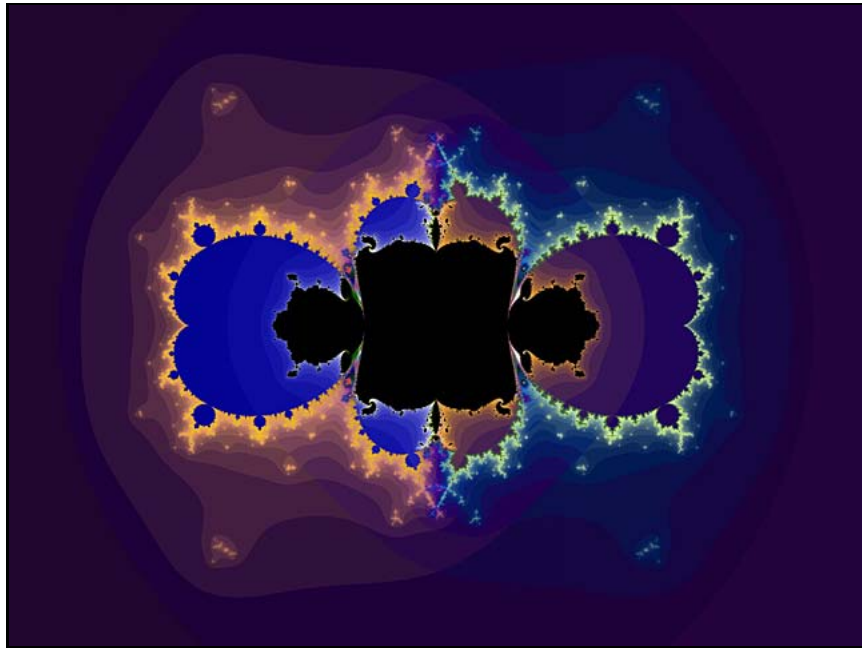after some exercises in algebra, can be rewritten in a form with one parameter

$p(z) = z^2 + c$,

any cubic polynomial,

$P(Z) = AZ^3 + BZ^2 + CZ + D$,

in like way can be rewritten in a form with two parameters

$p(z) = z^3 - 3a^2z + b$.

a = 0.6 + 0i.

Regarding the monster math to cubics, I've written a tutorial, ***CUBIC TUTORIAL.*** You can reach it from my index page. *Please read this tutorial carefully,* as all terms in this chaotic series regarding cubics refers to this tutorial.

Attached to this article is an example displaying b_{real}, b_{imag} when "a" is fixed to $0.6 + 0i$ (that is $a_{real} = 0.6$ and $a_{imag} = 0$). The image is drawn with respect to both critical points, $z = +a$ and $z = -a$, that is both M+ and M- are displayed, each in separate layers. In this article the parameter file is attached, so let's run it.

But what? The drawn image does not display the same fractal as the JPEG! Hehe. In the parameter file "a" is fixed to zero, that is we actually iterate $z \rightarrow z^3 + b$. The displayed fractal have the names *the Cubic Mandelbrot set, the Generalized Mandelbrot set for cubics, the Generic Mandelbrot set for cubics,* etc. Now dear reader we shall have a little lesson in using Stig's sub-module Cubic Parameterspace3. This sub-module is a better version than the old excellent Cubic Parameterspace and Cubic Parameterspace2. In our image

there are two layers named M+ and M-. Besides different gradients there is only one difference. When the layer M+ is selected, we at "M:" in the formula tab have M+ selected. That is we have drawn the set for which the first critical point   z = +a is the start-value. In a like manner when the layer M- is selected, we at "M:" in the formula tab have M- selected. That is we have drawn the set for which the second critical point z = -a is the start-value.

In this case when the parameter "a" is fixed to zero the critical points coalesce and the two set are actually the same, we have the generalized Mandelbrot set for cubics. To receive the fractal displayed by the JPEG, please type 0.6 in the box a_{real} under "constant Planes". NOTE: This must be done for both layers M+ and M-. In fact in this sub-module every change done in the fractal or location dialogue-boxes must be done equally to both layers M+ and M- in order to show up with mathematically correct images. For further facilities of Stig's module, see accomplished manual on his download page.

In Ultra Fractal the possibilities of multi-layering originally is intended for mixing different coloring-methods, mixing different kind of fractals and other kind of tricks making an inherent form of post-processing. Using multi-layered images when drawing parameter-spaces of higher polynomials using one layer with respect to each critical point, I will not call post-processing, but a method to get out more of the mathematical information. Let's have an example of this now.

Please select "Switch Mode" in order to view different Julia sets. Move the cursor around the parameter window and recognize how the Julia sets changes. In fact there are three cases.

1) When you move the cursor outside the fractal, the resulting Julia sets are Cantor dusts.

2) When you move the cursor inside the dark blue-orange or the blue part of the fractal, that is either M+ or M-, the resulting Julia sets have enclosed regions that are disconnected to each other.

3) When you move the cursor inside the black part of the fractal, that is the part of the image that is common to M+ and M-, the resulting Julia sets connected.

The black area is the set of those b-parameters, together with the fixed a-parameter, for which both the critical points z = +a and z = -a have bounded orbits. This area is a two-dimensional slice of the four-dimensional *cubic connectedness locus,* the full analogy for cubics to the Mandelbrot set for quadratics.

Play carefully :-)

Regards
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