17) The six perpendiculars slices intersecting origo

Because the standard iteration-formula for cubics, $z \rightarrow z^3 - 3a^2z + b$ has two complex parameters (a, b), the cubic parameter space is a four dimensional hyperspace made up of the axis a_{real}, a_{imag}, b_{real}, and b_{imag}. As human beings are not able to even imagine more than three spatial dimensions, this hyperspace has to be sliced up in 3- or 2-dimensional slices before viewing it.

A four dimensional hyperspace has four systems of 3-dimensional and six systems of 2-dimensional perpendicular slices. In cubics we therefore have the below 3-dimensional systems:

1) a_{real}, a_{imag}, b_{real}
2) a_{real}, a_{imag}, b_{imag}
3) b_{real}, b_{imag}, a_{real}
4) b_{real}, b_{imag}, a_{imag}

These can be studied with the 3D cubics and Raytracing modules of Stig.

The 2-dimensional systems are:

1) a_{real}, a_{imag}
2) a_{real}, b_{real}
3) a_{real}, b_{imag}
4) a_{imag}, b_{real}
5) a_{imag}, b_{imag}
6) b_{real}, b_{imag}

These can be studied with the sub-module ”Cubic Parameterspace3” of Stig. In this article we shall talk about the six two dimensional slices intersecting origo (the coordinate 0, 0, 0, 0), that is the slices were the two fixed coordinates are fixed to zero and the two others are plotted. These slices are all special both in respect to each other as well as to slices not intersecting origo. In order to make the relations more obvious, I’ve supplied the four axis in different colors. In these six images origo is the only coordinate that is common to all six planes. Note that in a four-dimensional hyperspace two planes can only intersect in a single point (if the planes not belong to the same three-dimensional cross-section). For example the figures a_{real}, a_{imag} and b_{real}, b_{imag} only intersect in zero. Now a brief description of the planes (In figures 1 - 6 the order of the planes are the same as in ”Cubic Parameterspace3”):

1) a_{real}, a_{imag}: The sets M+ and M- completely coalesces. The continent is a circle with the radius approximately 0.57735. (If you solve the equation $3a^2 = 1$, abs(a) is approximately 0.57735). On the border of this continent there are an infinite hierarchy of Mandelbrot buds. Their filaments are entirely built up of copies of the ordinary quadratic Mandelbrot set. There are four ”Mandelbrot heads”. Between two such heads the biggest one has a hierarchy of 4-armed stars according to the rules stated in article 10 ”The combinatorial rules of the filaments 1”. This means that the number of arms of the buds towards the ”Seahorse Valleys” increases according to the series 6, 8 10, etc instead of 5, 7, 9 towards the Seahorse Valley of the ordinary M set.

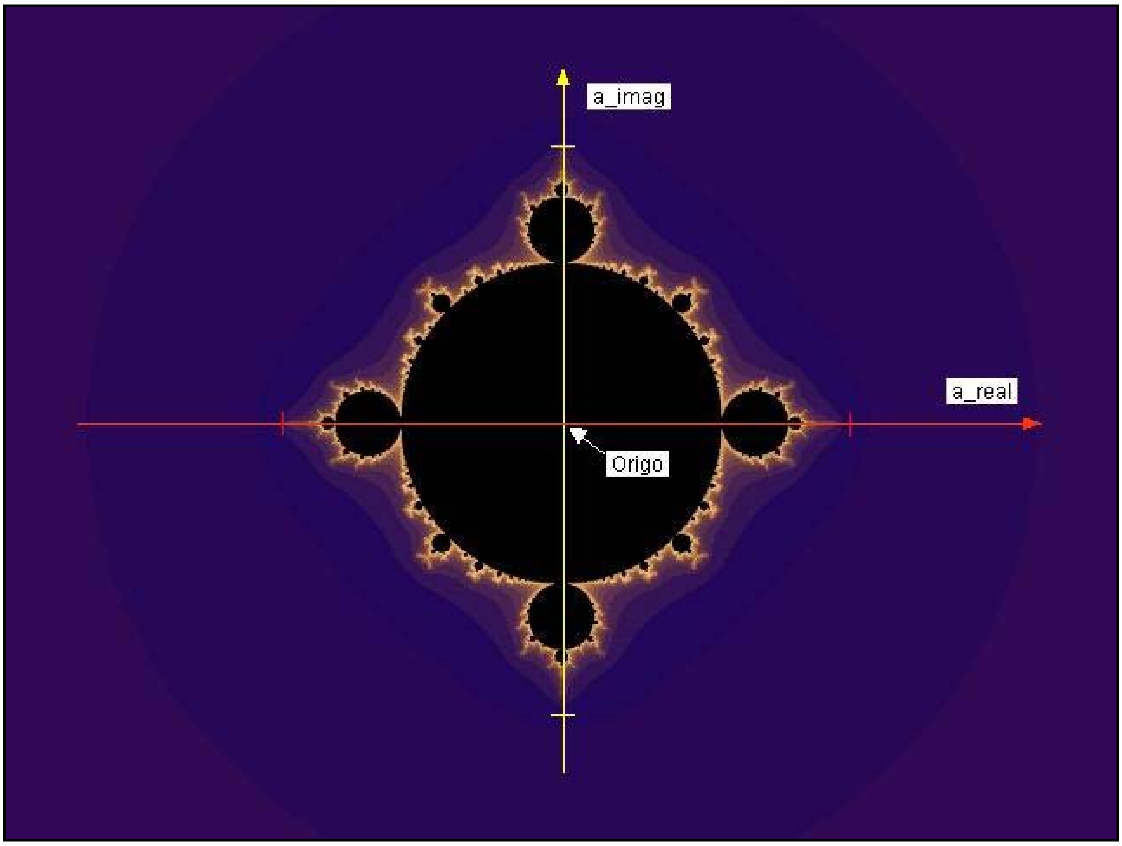

Fig 1. a_{real}, a_{imag}.

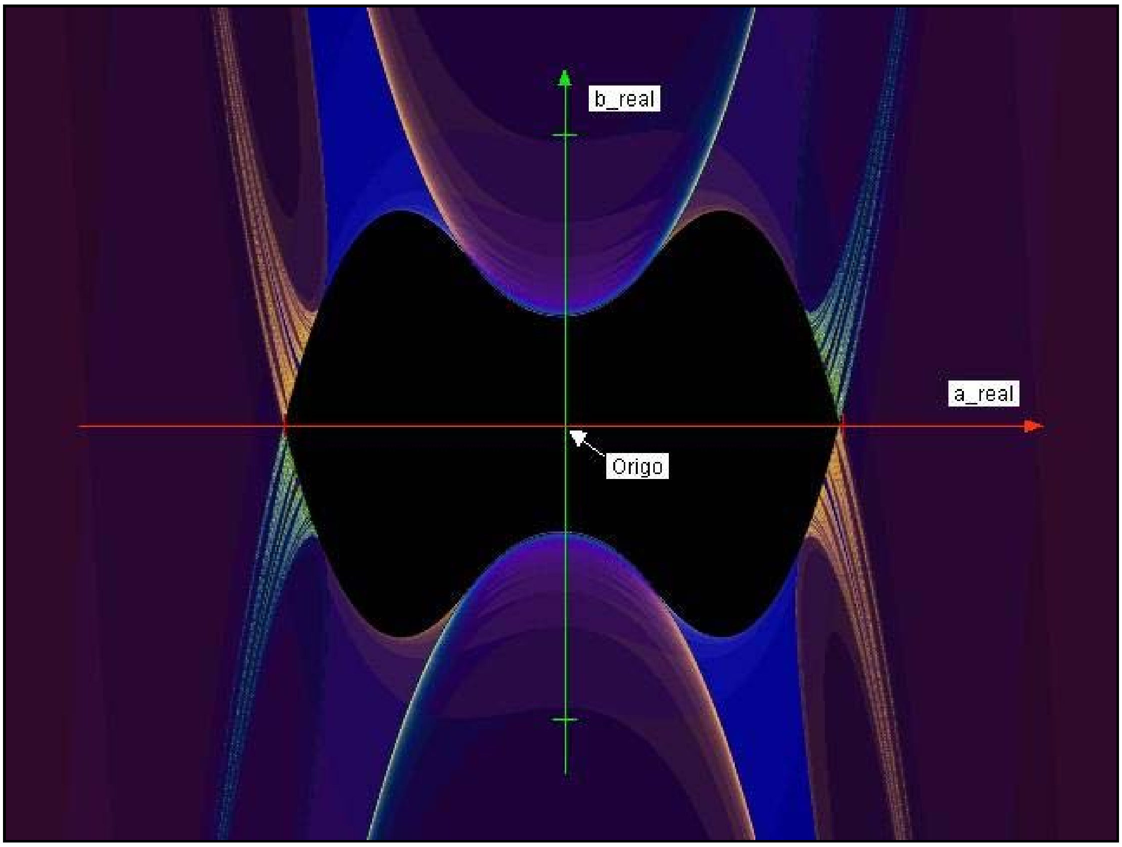

Fig 2. a_{real}, b_{real}.

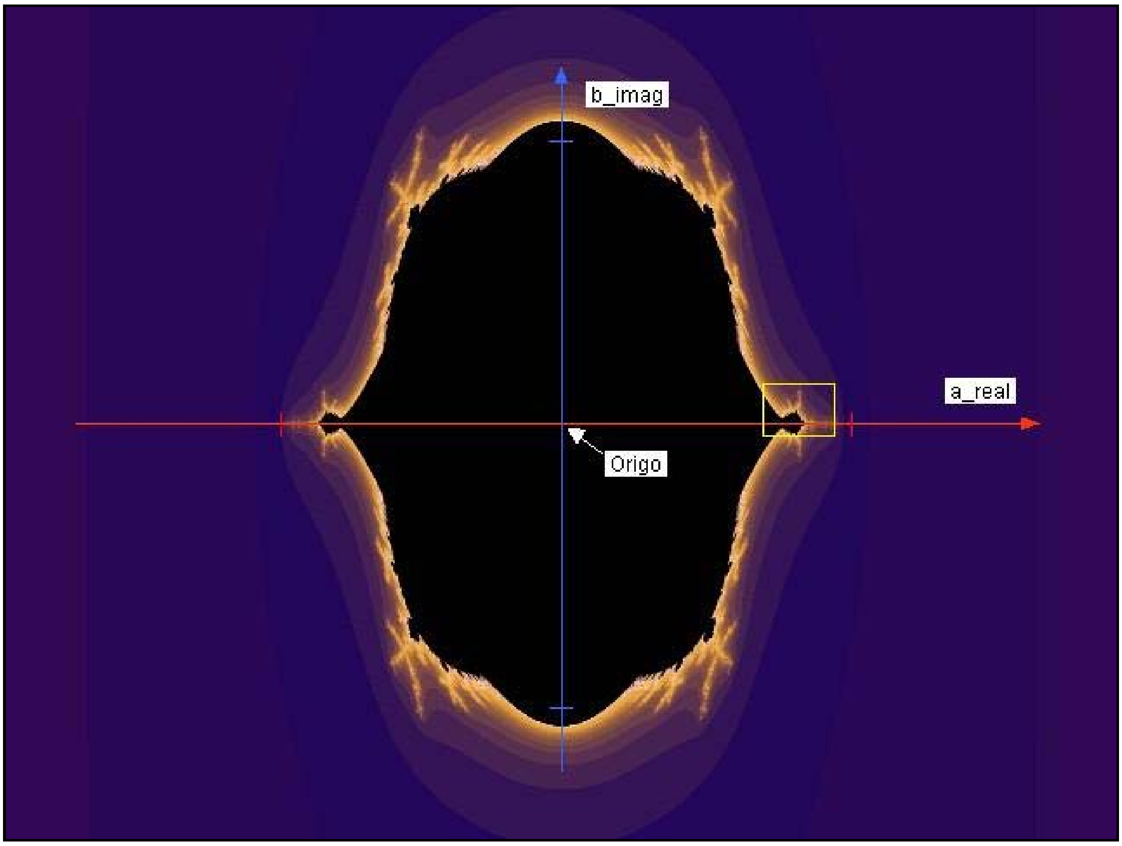

Fig 3. $\mathbf{a_{real}}$, $\mathbf{b_{imag}}$.

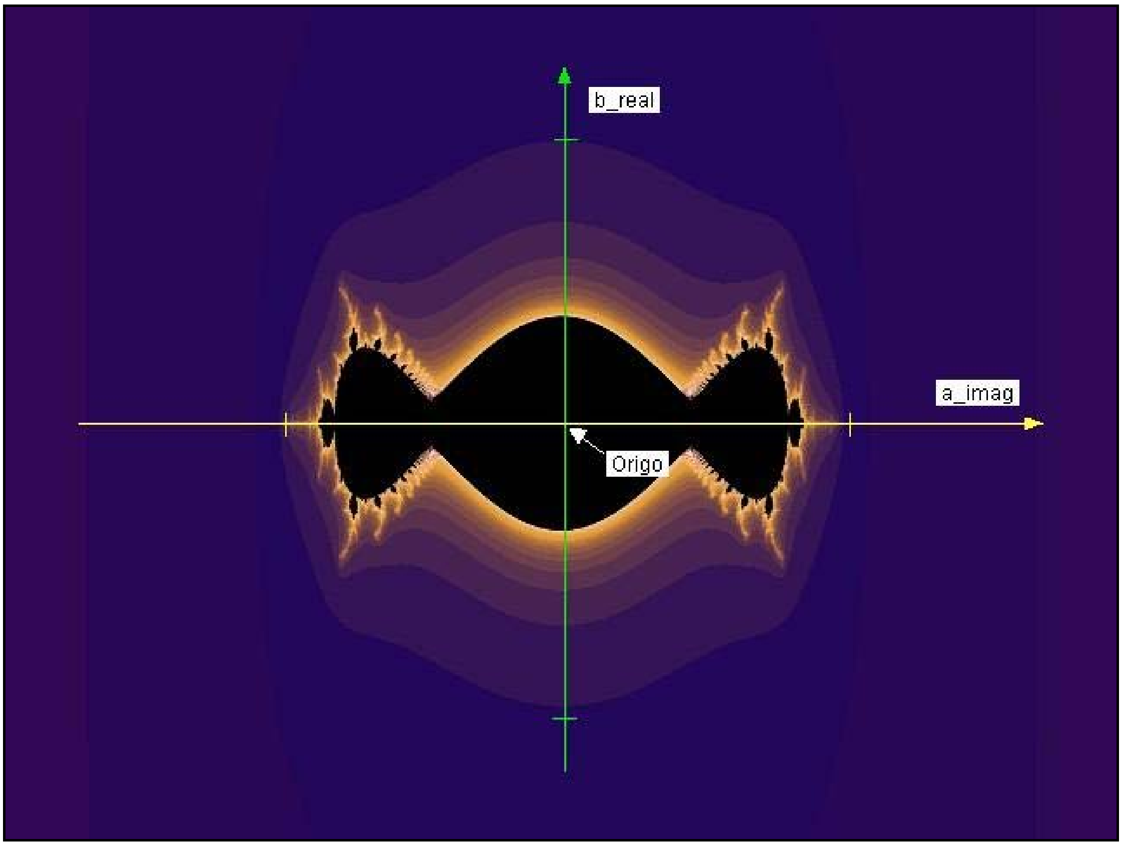

Fig 4. $\mathbf{a_{imag}}$, $\mathbf{b_{real}}$.

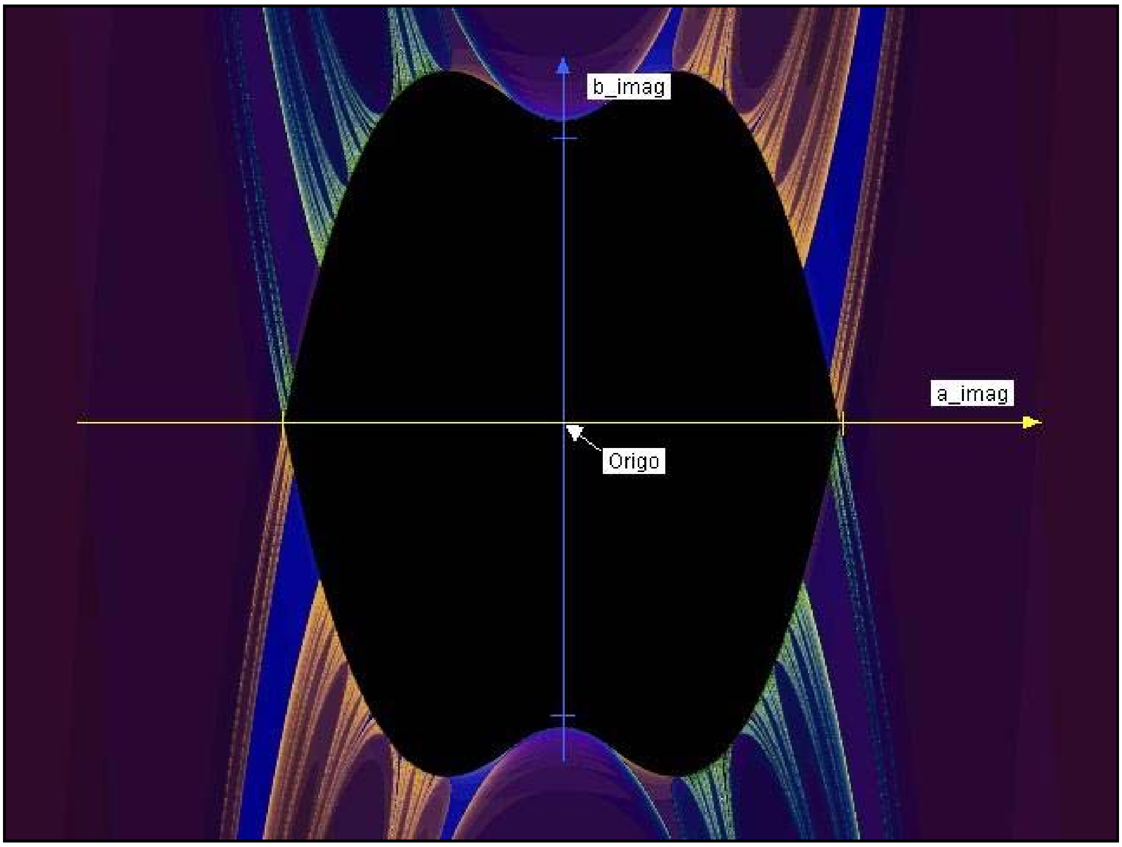

Fig 5. $\mathbf{a_{imag}}$, $\mathbf{b_{imag}}$.

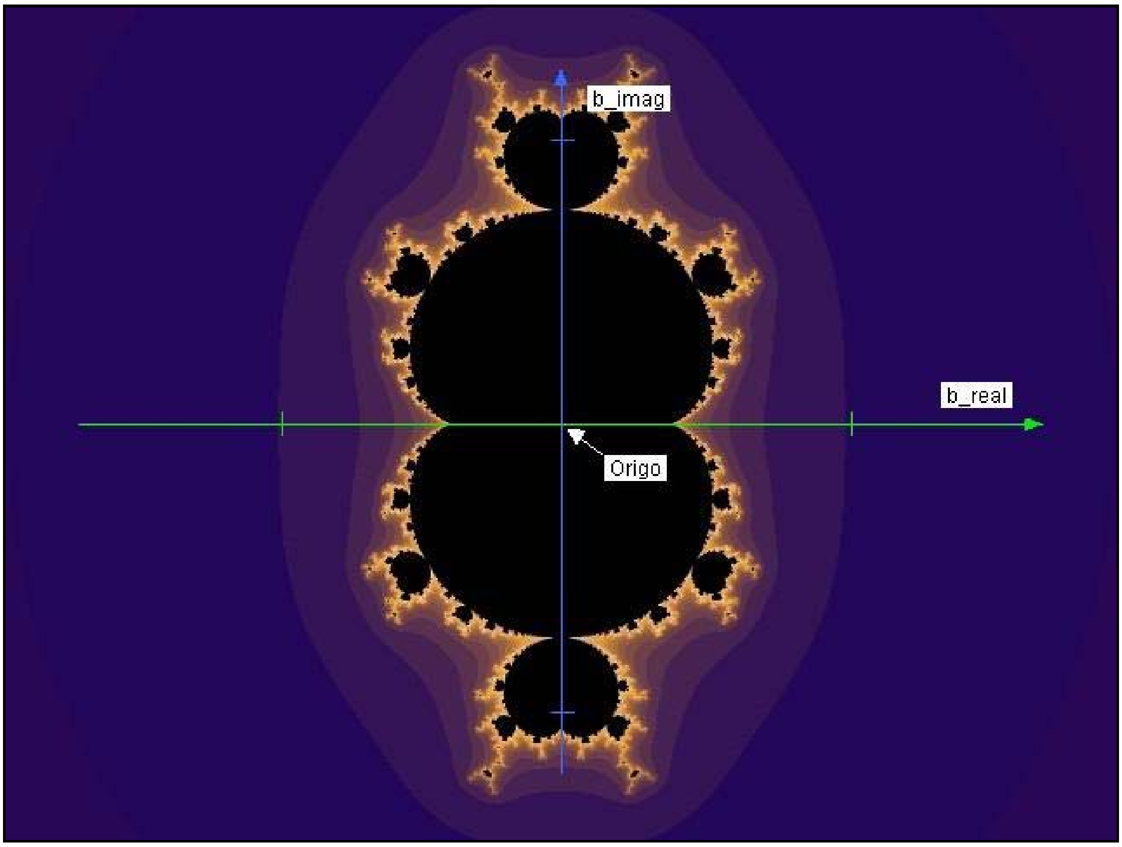

Fig 6. $\mathbf{b_{breal}}$, $\mathbf{b_{imag}}$.

2) a_{real}, b_{real} and a_{imag}, b_{imag}: In these two cases the respective border of C-locus is not a fractal. The ”towers”, belonging to either M+ or M-, are all ”side-views” of stapled Mandelbrot sets.

3) a_{real}, b_{imag} and a_{imag}, b_{real}: In these two cases M+ and M- coalesces. Parts of their respective borders are not fractal, parts are. The fractal parts of the borders have a little bit distorted Mandelbrot structures. The filaments, however (in a_{imag}, b_{real} only the spike-region), besides minibrots have one other type of component. In figure 3 (a_{real}, b_{imag}) there is a yellow rectangle announcing a short zoom-sequence a_{real}, b_{imag} Zoom1-3, figures 7 - 9. In a_{real}, b_{imag} Zoom3 you can see this component. It has threefold symmetry. This component has a prototype, the "Tricorn" (see figure 10, ”PrototypeTricorn”) and is generated by the dynamical process z -> $conj(z)^2$ + c, c = #pixel, z initiated to zero. If z = x + iy, conj(z) = x - iy.

4) b_{real}, b_{imag}: The sets M+ and M- completely coalesces and the critical orbits are identical. This well-known shape goes under the different names, ”the Cubic Mandelbrot set”, ”the generalized Mandelbrot set for cubics” ”the generic Mandelbrot set for cubics” etc. The filaments are solely built up of smaller copies of the cubic M set. The number of secondary decorations around each mini-copy instead of increasing according to the series 2, 4, 8, 16 etc increases according to the series 3, 9, 27 etc. That is 3^1, 3^2, 3^3 etc. In a future article some mysteries of this set will be described.

In the next article I will go on with some common features of the slices not intersecting origo.

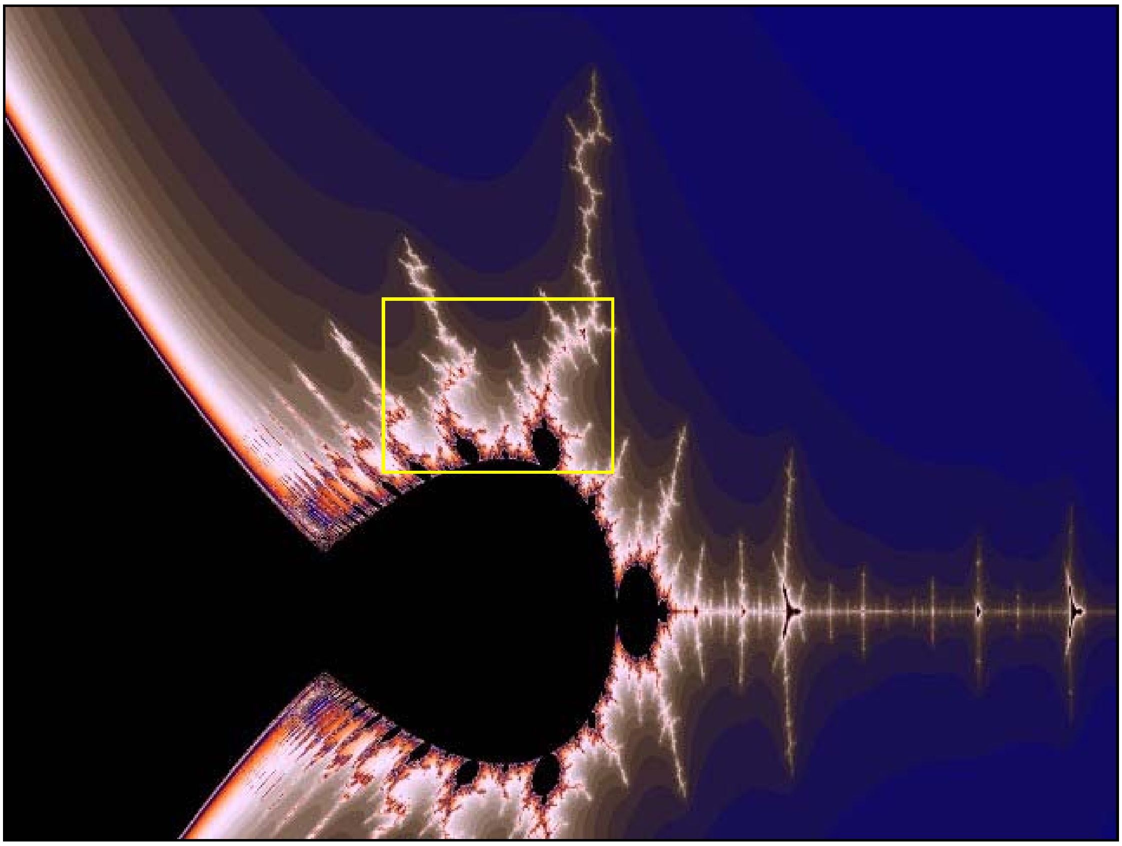

Fig 7. a_{real}, b_{imag}. Zoom 1.

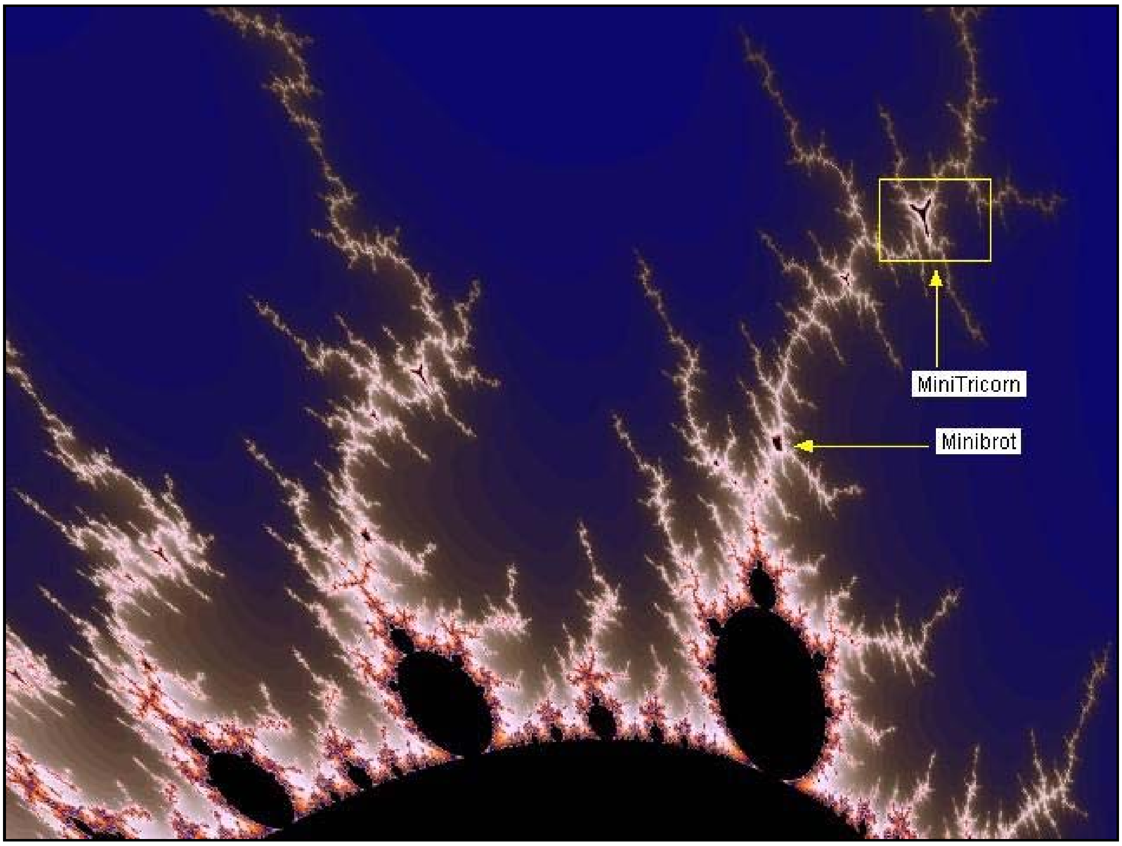

Fig 8. $\mathbf{a_{real}}$, $\mathbf{b_{imag}}$**. Zoom 2.**

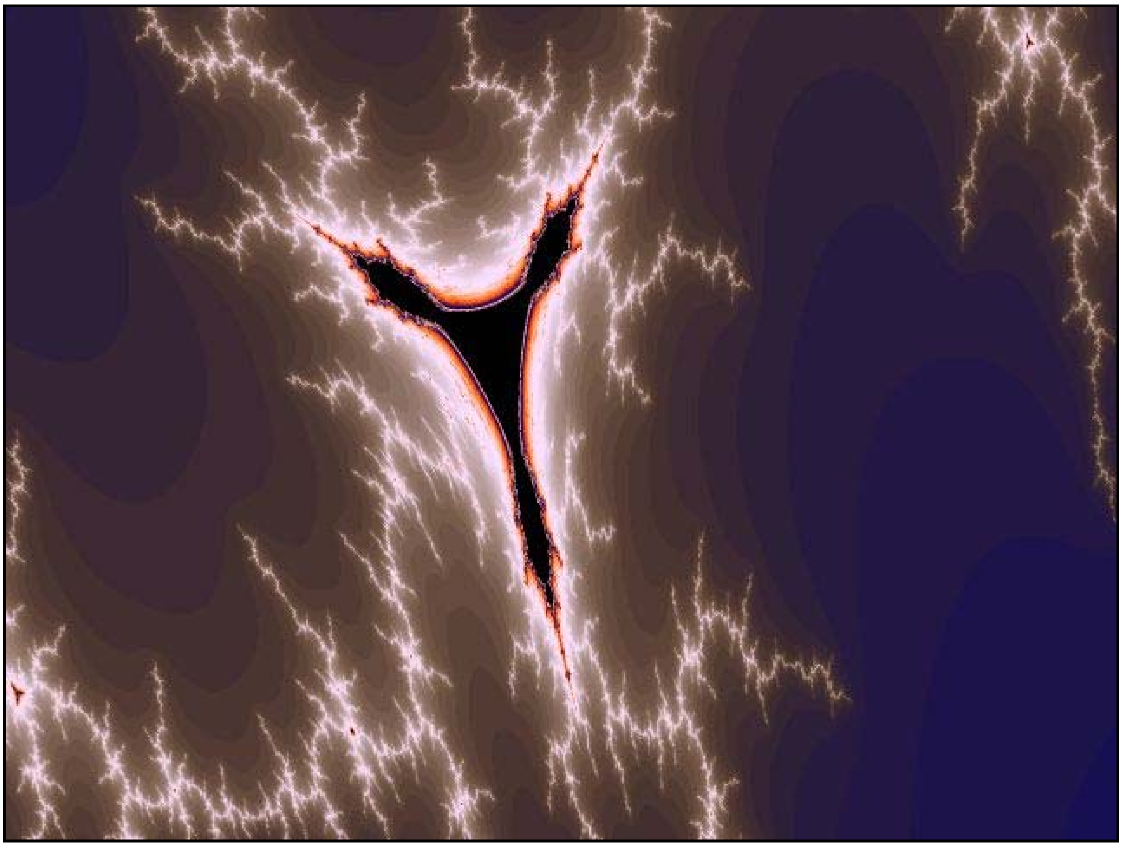

Fig 9. $\mathbf{a_{real}}$, $\mathbf{b_{imag}}$**. Zoom 3.**

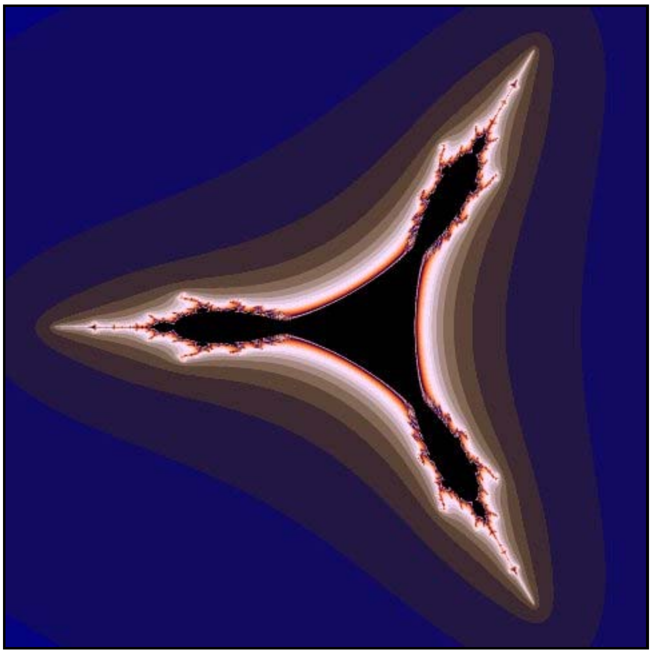

Fig 10. Prototype Tricorn.

Don't forget my ”Cubic Tutorial” and ”Pictures from Cubic Parameterspace” reachable from my index page.

Regards
Ingvar