## 19a) Dual critical points manual

This is a short manual how to draw 2D-images of cubic parameter space with respect to both critical points, z = +a and z = -a, using Stig's sub-module "Cubic Parameterspace3". In this example we will draw the  $a_{imag}$ ,  $b_{imag}$  after having moved 0.2 units in  $a_{real}$ -direction and 0.2 units in  $b_{real}$ -direction. Note: Since this is an exercise no parameters are posted. The dear readers are expected to come up with the below images themselves following the instructions below.

1) Select "New" and then "sp3" and thereunder "Cubic Parameterspace3". Increase the max iteration to 1000 and, under "outside color", increase the color density to 4 (the last two steps are not essential). Then you will obtain figure 1.



2) Change "PlottedPlane" to from " $a_{real}$ ,  $a_{imag}$ " to " $a_{imag}$ ,  $b_{imag}$ " and you will receive figure 2.

3) Under "Layers", click "Background" and rename it to M+. This makes no difference for the image. It's only for pedagogical reasons denoting that "z" is initialized to +a and thus the set M+ is drawn.

4) Add "New Layer" by clicking the document with "+" to the right.

5) Rename this new layer M- This is also for purely pedagogical reasons in order to show that, in next step for this layer, "z" is initialized to -a and thus M- will be drawn.

6) When the M- under Layers is selected turn the Opacity from 100% to 50%. Then go to "M" and change from M+ (which is there by default) to M-. You will now receive figure 3. If you move the Opacity control in left direction, you will see more of M+, and if you move it in right direction you





will see more of M-. The area that remains totally black is the set is common to M+ and M-. That is a 2D-slice of *Cubic Connectedness Locus (CCL)* the full analogy to the Mandelbrot set for cubics.

7) Under "Constant Planes", type 0.2 in the line "a-real". Very important. You must do this for both M+ and M-. In fact, as soon as M+ and M- being established, all future changes under "Formula tab" as well as under "Location tab" must be done equally for M+ and M-. If not you maybe will obtain a pretty picture, but this picture does no longer represent a correct view of cubic parameter space, but two different views mixed together. However it's strongly recommended to use two different gradients, one for each layer, making it easy to see what is solely M+ and solely M-. I leave to the diligent reader to construct these (or you can take my gradient-settings from any of my parameter files in the previous article). Another way is to move the Opacity control as shown above. The resulting image is figure 4. In this case we can consider "a<sub>real</sub>" as the axis coming out from your computer screen giving you a punsch on the nose. Typing 0.2 in this box means you move 0.2 units from origo towards your nose.

8) Still under "Constant planes", type 0.2 in the line "b-real" for both M+ and M- and you will achieve figure 5, our final image. The axis  $b_{real}$  is the axis that is at the same time perpendicular to all the other three axis'  $(a_{imag}, b_{imag}, in this case the axis' building up our screen, and <math>a_{real}$ , giving you a punsch on the nose). That means it is pointing into the fourth spatial dimension. This typing means you move 0.2 units from origo into the fourth dimension.



**Some observations**: For a brief description of the special slice displayed in figure 3, see article 17 (The six perpendiculars slices intersecting origo). The symmetry between M+ and M- in this image and in figure 4 is along a diagonal. However in the final image, figure 5, no symmetry exists. M+ is entirely inside M-. My experience from computer experiments tells me there is symmetry in all perpendiculars 2D-slices with the following exception: In  $a_{real}$ ,  $b_{real}$  and  $a_{imag}$ ,  $b_{imag}$  there is no such symmetry if the slice is made after having moved from origo along both the non-plotted axes.

**Punishment:** Now after you have achieved the last image (figure 5) as a punishment I encourage you to do the Switch Mode exercise in Article 16 (Introduktion to Cubics) and study what happens to the Julia set when you move the cursor in E-, B-, and C-locus.

In the next article I will describe the features that are new in Cubic Parameterspace3.

Regards Ingvar