

## 22) Rotations in 4D-space

This time we shall first make a little zoom in the b-plane ( $b_{\text{real}}, b_{\text{imag}}$ ) when  $a = 0.6 + 0i$  ( $a_{\text{real}} = 0.6, a_{\text{imag}} = 0$ ). In "Cave-zoom2" (figure 3) we are fully inside M-, so we have better to delete that layer in order to speed up the calculations. Let's also change the gradient to the old UF-default. Our goal "Cave-zoom5" (figure 6) is the same motive as "the Birth of the Mystery in the Cave" displayed in the cubic part of my web-site. The center of this image belongs to the cardioid of a minibrot. Now, like we did in the previous article, we shall have a look at the environment around, in this case, the spot situated in the cardioid when we rotate to first the Plotted Plane 1,  $a_{\text{real}}, a_{\text{imag}}$ , thereafter to the remaining four perpendicular planes. I recommend the dear reader to temporally save the results of the rotations in order to perform further investigations. Below is a step by step description of how to perform this using Cubic Parameterspace3:

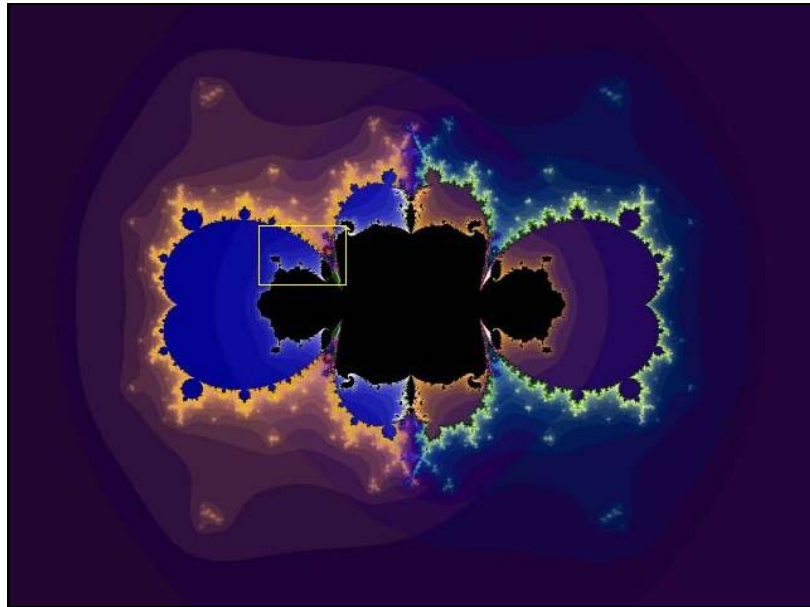


Fig 1 Cave-start.

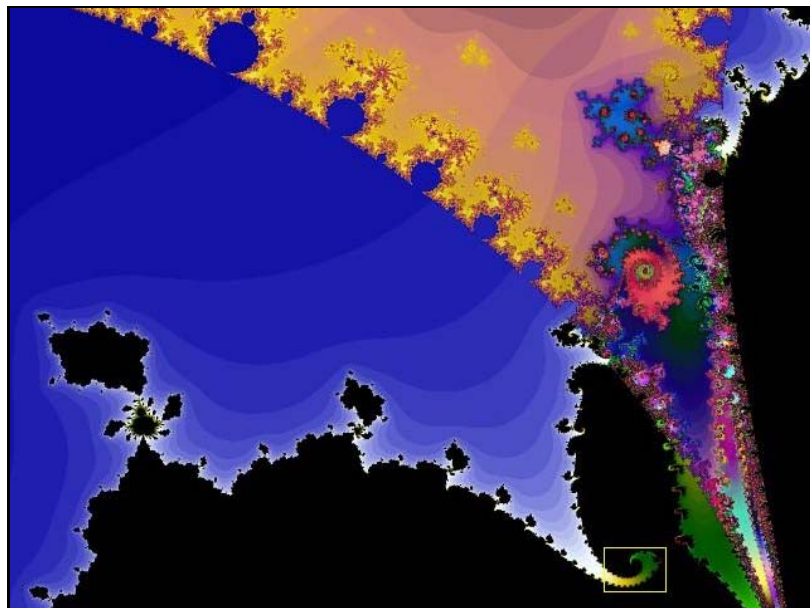
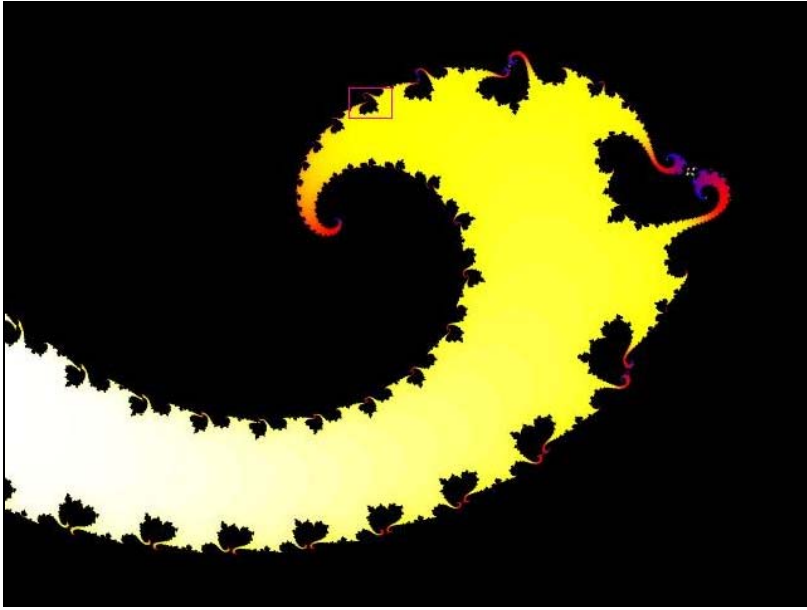
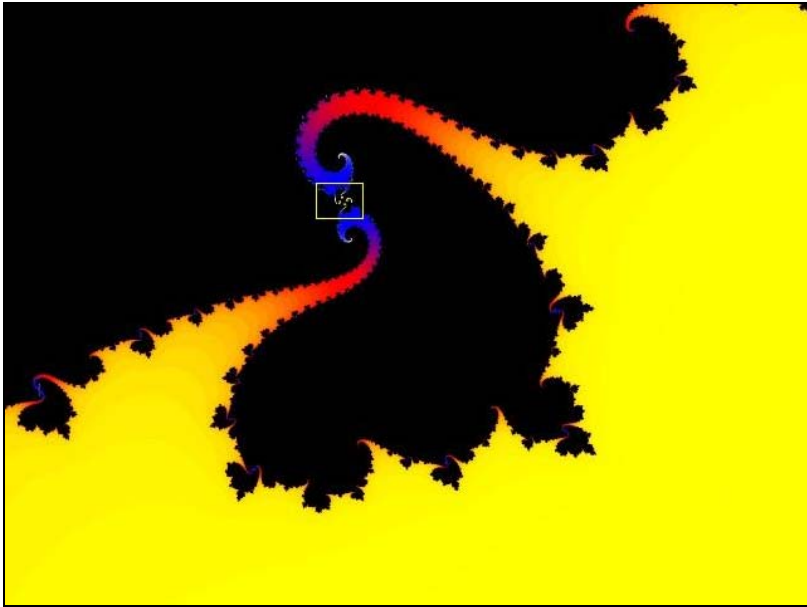


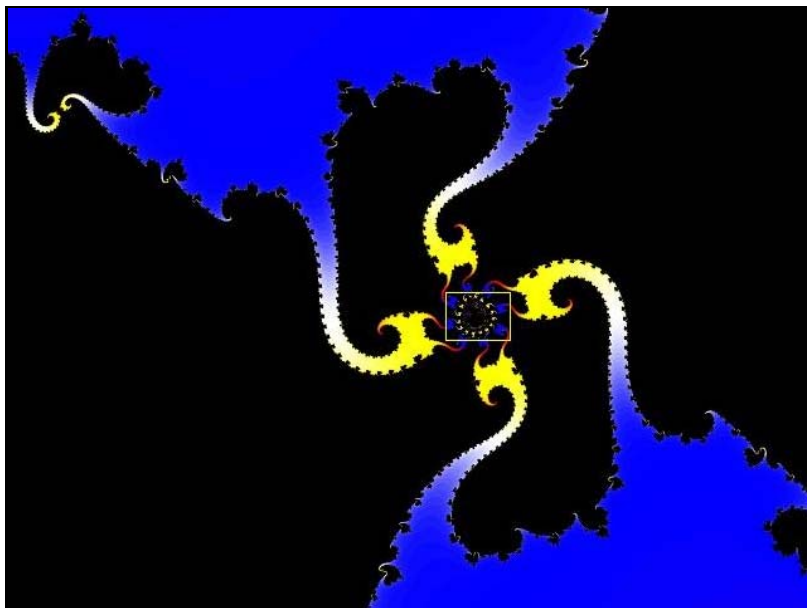
Fig 2 Cave-zoom 1.



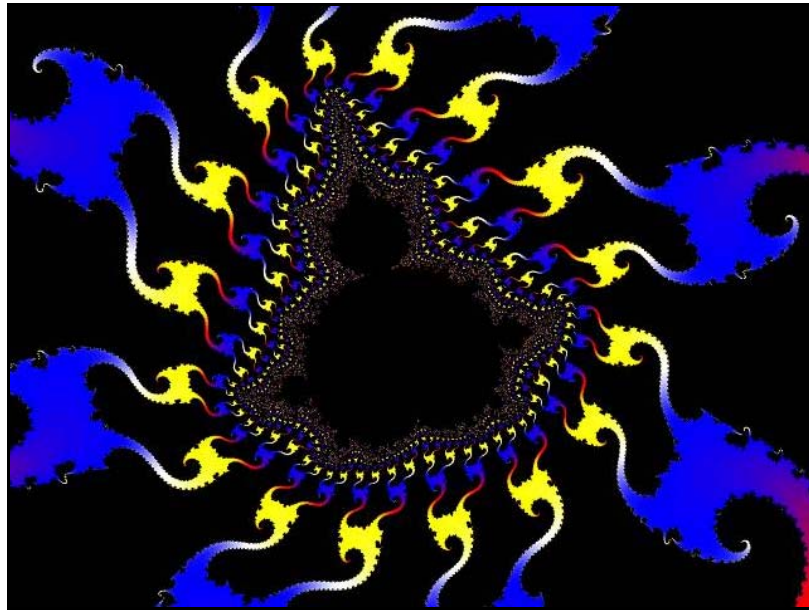
**Fig 3 Cave-zoom 2.**



**Fig 4 Cave-zoom 3.**



**Fig 5 Cave-zoom 4.**

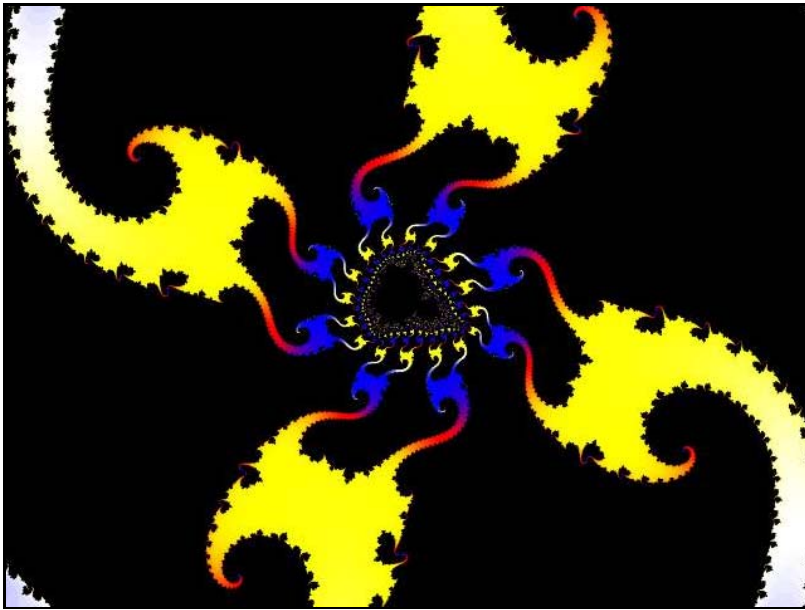


**Fig 6 Cave-zoom 5,  
plane 6.**

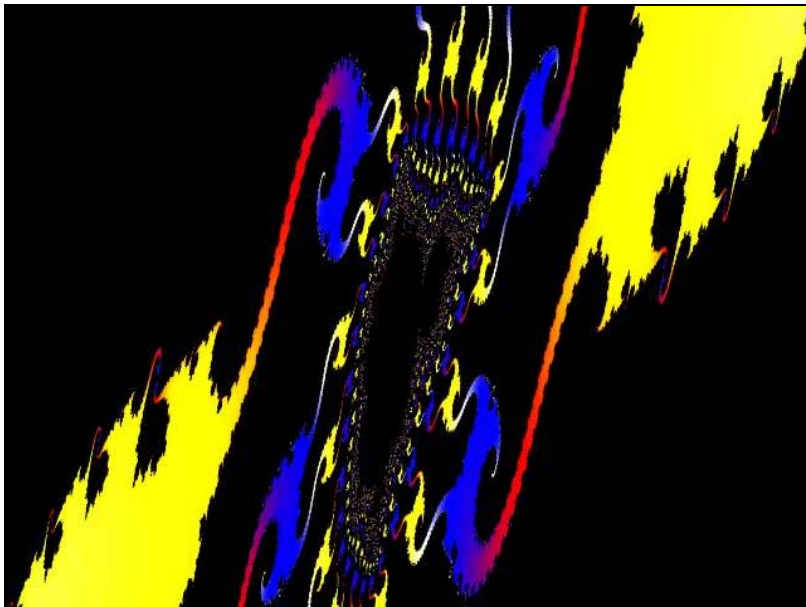
1) Select the Location tab and click the pencil at the coordinate you want to rotate around (if another than in the current center). When you move the cursor through the image, you will see the actual position of the cursor displayed as center coordinate. Now, if you click a spot in the current image, UF will recalculate the image with the clicked coordinate as center coordinate. In our case we will make attention of the spot which is already in the center, so we disable the pencil (click the pencil again). NOTE: The names "Center (Re)" and "Center (Im)" below are sometimes illogical since "Center (Re)" not always represent a real axis and "Center (Im)" not always represent an imaginary axis. In fact "Center (Re)" should be taken as the horizontal- (x-) axis and "Center (Im)" should be taken as the vertical- (y-) axis.

2) Now go to Formula tab, "Planes". Unmark the little box "Disable plotted planes", the result being the fields "a-real" and "a-imag" are no longer dimmed so you can type values in these boxes. Now copy Center (Re) and paste this value to "b-real" under the Formula tab and thereafter copy Center (Im) and paste this value to "b-imag". As b-real, b-imag is the plotted plane the values typed in b-real and b-imag make no sense. UF will recalculate exactly the same image. Let's call it "Cave-zoom 5 - Plane 6". But now we have the entire 4D-coordinate typed in a-real, a-imag, b-real, and b-imag, and this is necessary when we will do our rotations. A very practical preparation is to type the whole 4D-coordinate into a notepad-document as in the attached one.

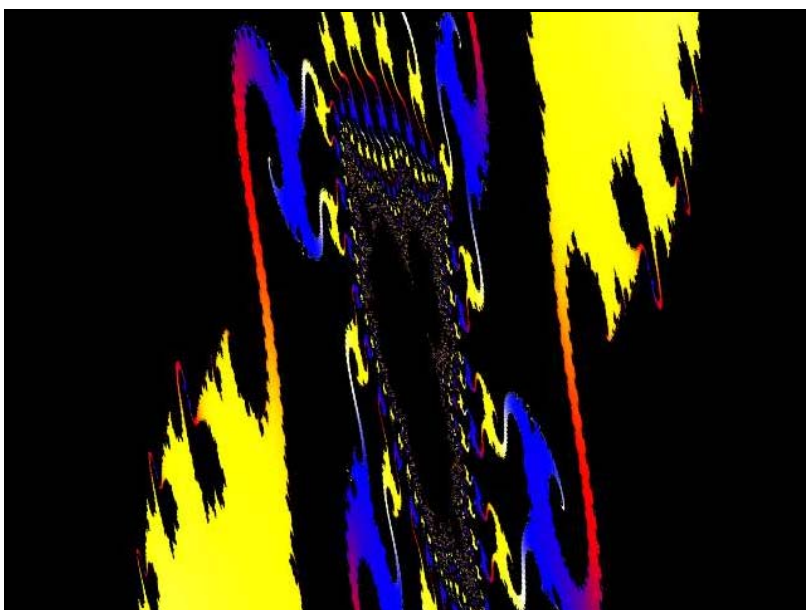
3) In this step we will change to Plotted Plane "1).a-real, a-imag", so let's do it (figure 7). Having done so, you must copy and paste the  $a_{\text{real}}$  value to Center (Re) and  $a_{\text{imag}}$  value to the Center (Im) (that's 0.6 respective 0 so you can simply type them). This can be done from the respective boxes under Formula tab, but having the 4D-coordinate in Notepad it's much easier to do it from this. Now you will obtain "figure 7. Cave-zoom 5, plane 1". If you do a little zoom towards the center in this motive you will see almost the same motive as "figure 6. Cave-zoom 5 - Plane 6" (despite from the angle), and the only spot these two images share is a spot in the cardioid of each minibrot! Now we have performed a kind of double-rotation only possible to perform in 4D-space.



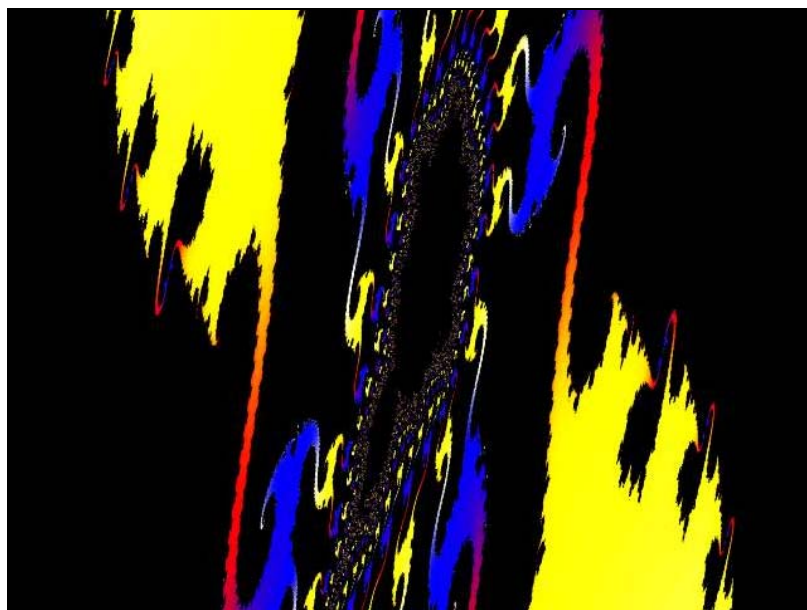
**Fig 7 Cave-zoom 5,  
plane 1.**



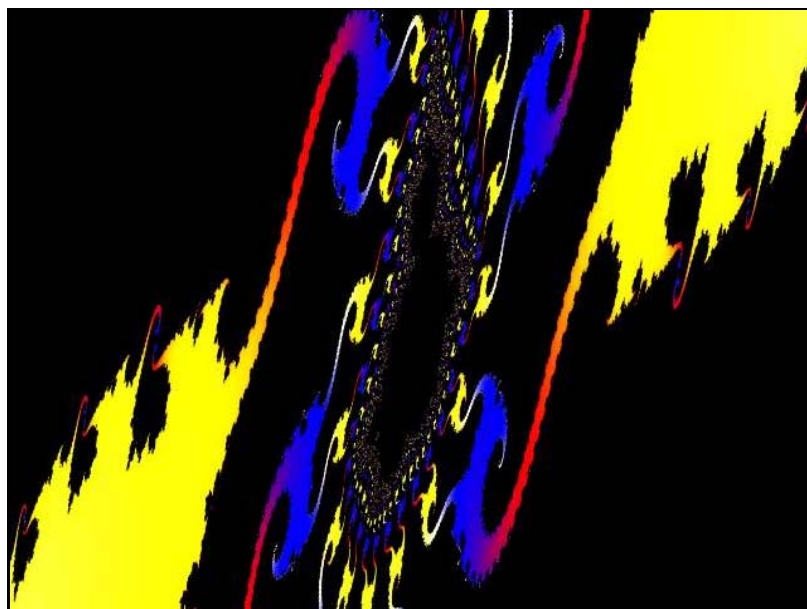
**Fig 8 Cave-zoom 5,  
plane 2.**



**Fig 9 Cave-zoom 5,  
plane 3.**



**Fig 10 Cave-zoom 5,  
plane 4.**



**Fig 11 Cave-zoom 5,  
plane 5.**

4) Now we want to see the same spot in Plotted Plane 2: a-real, b-real (figure 8. Cave-zoom 5, plane 2), so let's change to this plane and copy the value of  $b_{\text{real}}$  to the Center (Im) {the value of  $a_{\text{real}}$  is already in Center (Re)}. It's always the values belonging to the plotted axis that should be copied to Center (Re) and Center (Im).

5) Do the same operations in order to see the remaining three planes by altering "Plotted Plane" (figures 9 - 11). When you do so you must copy the value from the first plotted axis and paste it to Center (Re) and then copy the value from the second plotted axis and paste it to Center (Im). Here it will be advantage to have the whole 4D-coordinate in a Notepad-document.

Note: It does not matter in which order all these operations would be performed.

**General observation:** In all planes we have similar motives in different angles, but in the Plotted Planes other than 1) a-real, a-imag and 6) b-real, b-imag the motives are rather tilted. Now having saved the results of your deeds

it would be interesting for you to do successive out-zooms in order to see the parent fractals (with eventually additions of an extra layer for M-). Then note that all motives are situated in a cave like in our original in zooming, but the parent fractals are all quite different.

**Question:** All the six perpendicular slices, as we have seen by the above practice, share at least the 4D coordinate:

$$* a_{\text{real}} = 0.6$$

$$* a_{\text{imag}} = 0$$

$$* b_{\text{real}} = -0.417189212099936605$$

$$* b_{\text{imag}} = 0.132251442324604608$$

However some Plotted Planes, for example the  $(a_{\text{real}}, a_{\text{imag}})$  and  $(b_{\text{real}}, b_{\text{imag}})$  have no axis at all in common. This would mean that these planes only intersect in a single spot! However two distinct planes, if they intersect, can only do so in a line?

**Answer:** That's true in a three dimensional space. The four dimensional hyperspace, however, have properties which are quite impossible for human beings to even imagine. One of those properties is that two planes, if they intersect, can only do so in a single point, as we together have shown by the above practice. Other absurd properties of a 4D space are for example:

- You are not able to knot a knot in 4D-space. Every knot you move into this space becomes unknotted. However you can make knots of areas!

- In 4D-space you can turn a football inside out without perform any cuttings.

- If you moved a suitable distance in the fourth dimension you should see every closed room in the house you are dwelling in, including you even would see the inner organs belonging to the people in this house. If you didn't lose you sense, you even would be able to move in and out in closed rooms.

- At least: If you find it complicated to perform the above described rotations, you shall know that this was much more difficult to do with the first module "Cubic Parameterspace". And don't get lost in the fourth dimension.

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Regards  
Ingvar