23) Every Connectedness Locus is connected

(Note: For correct views of some of the images, set the size of the images to 100 or 125%)

In every parameter space the *connectedness locus* (C-locus) is the set of those parameters which give rise to connected Julia sets. Hence the name. But is C-locus for a polynomial of any degree itself connected? The answer given by the great mathematicians is yes. I do not understand their complete evidence, but let's have a look at their argumentation regarding the C-locus of quadratics, that is the Mandelbrot set.

Figure 1, "ContinousMandel", shows the whole Mandelbrot set. There is a refinement of the Level Set Method (LSM) coloring according to

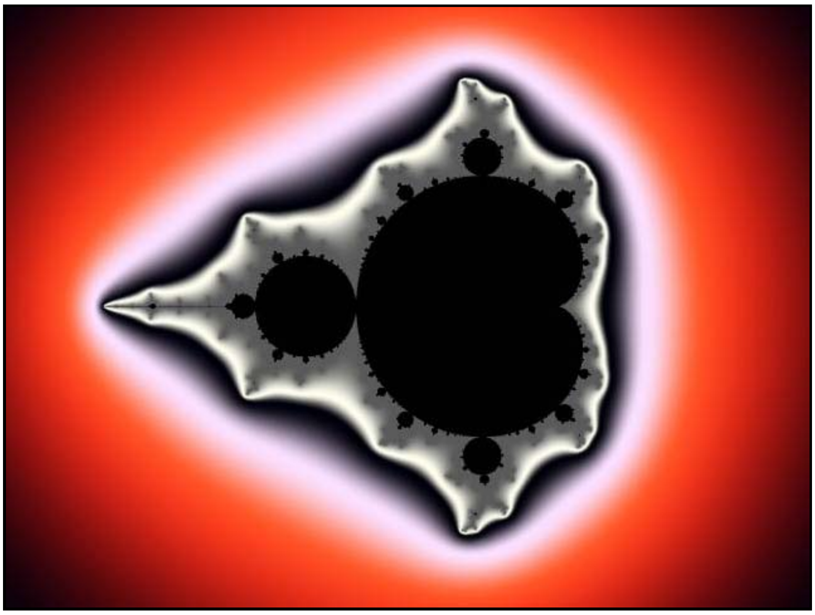

Fig 1. Continous Mandel.

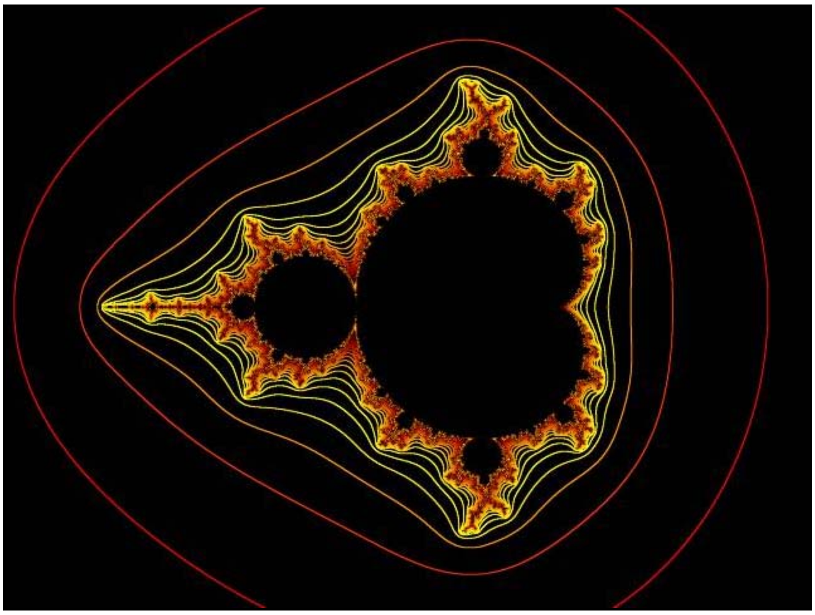

Fig 2. Equapotential Mandel.

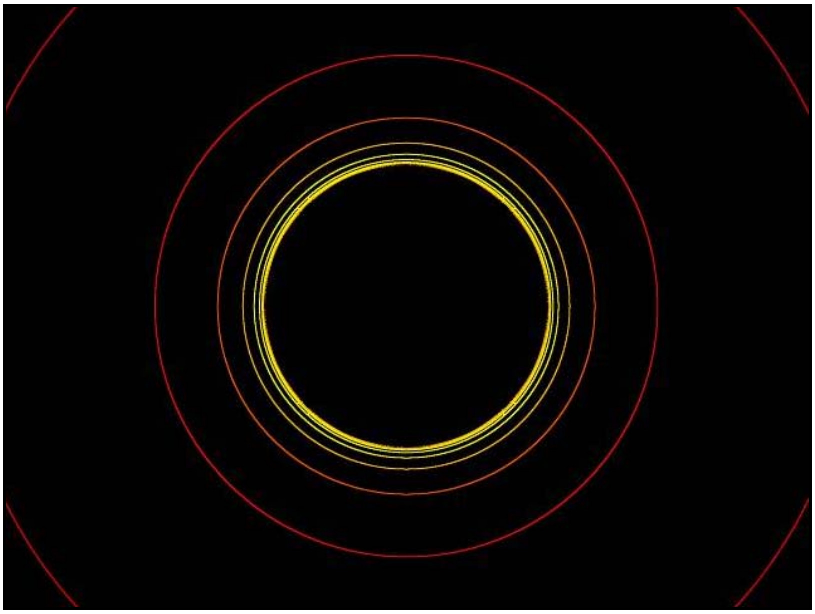

Fig 3. Unit Disc surrounded by equapotential curves.

Sorry Folks,

There are NO human beings who are able to imagine any four dimensional object or the curved spaces that may approximate them.

Fig 4. Curved Spaces around CCL.

Sorry Folks,

There are NO human beings who are able to imagine a hypersphere. A hypersphere is the three dimensional "surface" of a four dimensional ball. This means that you will return back after moving straight away in any of the three spatial dimensions.

According to some teories of the universe the universe is a very big hypersphere. This depends on the denseness of matter.

Fig 5. Hyperspheres around the Unit hypersspere.

Fig 6. Thorns.

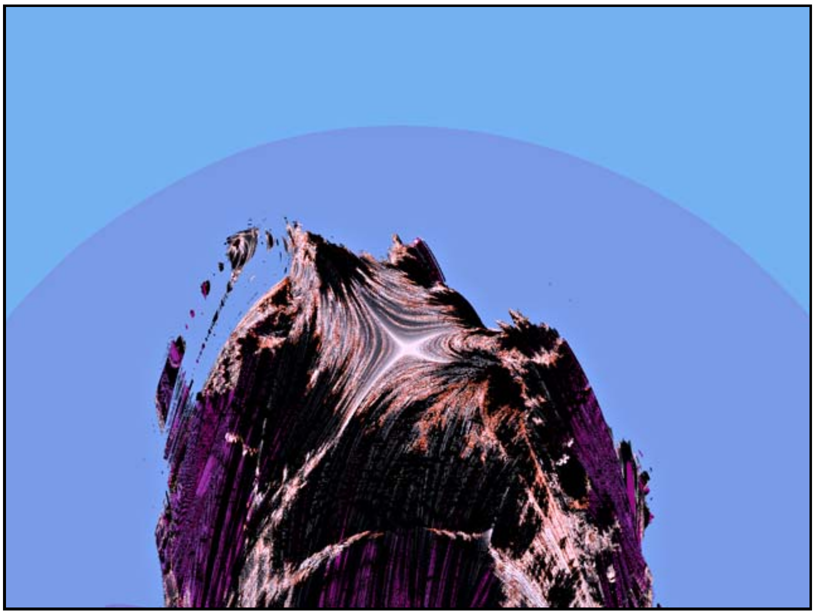

Fig 7. Asteroid. The set which is the common part of the red and white set in figure 6.

iterations, called ”Continuos Potential Method” (CPM). I do not know the math behind this method, but the idea is to achieve a continuos slope around the set. Then every nuance of color around the set represents a certain escape-rate. These escape-rates can be represented as lines, as shown in figure 2, ”EquipotentialMandel”, (here technically generated by ”MandelLevel”). Now the great mathematicians have shown that these level-curves behave in the same manner as the circles around the closed unit disc, shown in figure 3 ”UnitDisc”, (here technically generated by JuliaLevel for the Julia set of z -> z^2). Hence the Mandelbrot set is connected. Now these mathematicians have extended this result to the C-locus belonging to polynomials of any degree. So regarding cubics they have shown that the curved spaces, denoting the fastest escaping critical point, around the four-dimensional *cubic connectedness locus* (CCL) behave like the hyperspheres around the unit hypersphere. Figure 4, ”Curved spaces around CCL”, illustrates how the curved spaces more and more approximate the four-dimensional CCL in the same way as the curved lines more and more approximates the M set. Figure 5, ”UnitHypersphere”,

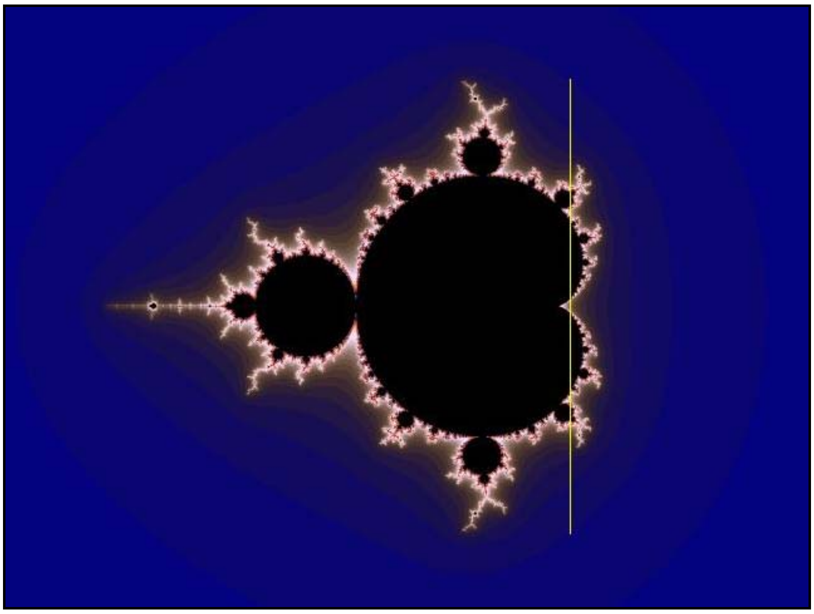

Fig 8. Sliced Mandel.

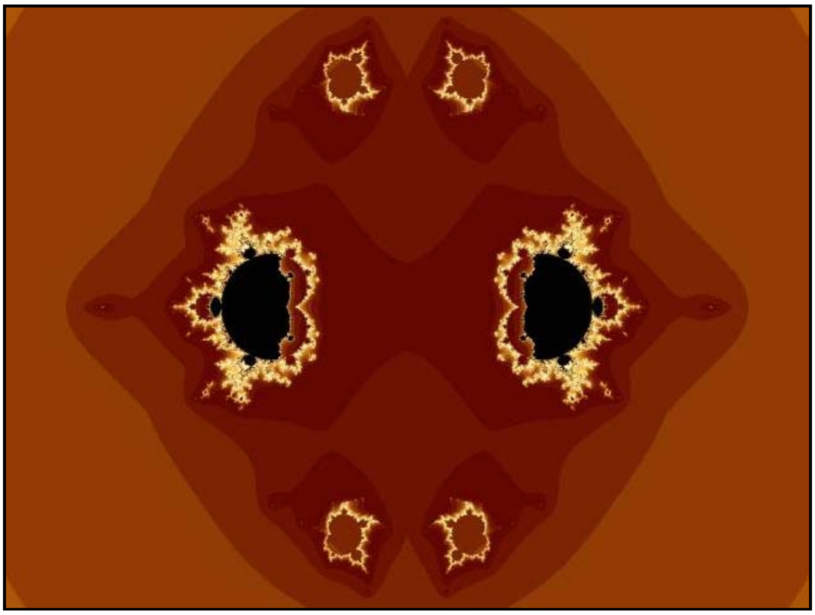

Fig 9. Disconnected 2D.

illustrates the hyper spheres around the unit-hypersphere. Now let's have look at a 3D-slice of cubic parameter space.

In figure 6, ”Thorns”, produced by the raytracing module of Stig, a slight magnification of a 3D-slice is done, the axis b_{real} pointing into the fourth dimension, the 3D-slice being done at $b_{real} = 0$. The red thorns display solely M+ and the gray thorns display solely M-. Where the thorns reach the walls of a virtual room they are cut off, and their slices are copies of the ordinary Mandelbrot set. The C-locus is the locus that is common to M+ and M-.

The image ”Asteroid” (figure 7) is also drawn with the old 3D-module displaying exactly the same coordinates. However here only the set that is common to M+ and M-, that is CCL, is displayed.

Question: In the left upper part of this image there are small blocks separated from the main asteroid. How can then this set (cubic connectedness locus) be connected?

Answer: This image is only a three-dimensional cut-off from the four-dimensional main set. In fact the set is connected in the fourth dimension. To illustrate this, look at figure 8, ”Sliced Mandel”. In this image a yellow line is drawn in order to make a one dimensional slice. In this slice the Mandelbrot set shows up in at least four disconnected pieces, but if you take the second dimension into account, you will see that the M set is connected. OK, you can't know that the filaments are connected. There you have to thrust the great mathematicians.

Similar phenomena may occur in some 2D-slices. If you look at figure 9 ”Disconnected2D”, the C-locus shows up in two main pieces. However if you move along the axis pointing into the third dimension in a suitable direction, in this case if you type zero into the Formula- box a-real, you will obtain a connected C-locus. In this image CCL is drawn with ”SetBorders” enabled, meaning that also the border of B-locus is displayed.

Sense moral: Although a slice of connectedness locus for higher degree polynomials may show up disconnected, they are connected in other dimensions.

Regards,

Ingvar