24) Mysteries in the Cubic Mandelbrot set

(In order to see the text in some illustrations, turn the size temporally to about 125%)

In the previous article we have seen that although the connectedness locus for polynomials of any degree is connected, particular slices sometimes are not. Now the C-locus for any polynomial have a special slice generated by the formula z -> z^d + b, d = 2, 3, 4 etc, ”b” plotted and ”z” initialized to the critical point ”0”. For d = 2 we obtain the classical Mandelbrot set. For d = 3, a special slice of $z^3 - 3a^2z + b$ when ”a” is fixed to zero (see Article17), we obtain the ”Cubic Mandelbrot set”, ”generic Mandelbrot set for cubics”, ”generalized Mandelbrot set for cubics” etc, the names are somewhat different. Before starting our journey to some mysteries in the cubic Mandelbrot set, we shall keep in mind that the great mathematicians have shown that the generalized Mandelbrot set for any degree is connected. Now let’s start our zoom.

Like the ordinary M set, the generic sets have filaments constituted of mini copies of the whole respective generic set. We zoom in one of the largest one at the upper left horn. Note that as the number of secondary decorations around minibrots of the ordinary M set increases according the series 2, 4, 8, 16 (that is 2^1, 2^2, 2^3, 2^4 etc) when approaching the minibrot, the number of decorations around mini copies of the Cubic Mandelbrot set increases according the series 3, 9, 27, 81 etc (that is 3^1, 3^2, 3^3, 3^4 etc). This is cleary

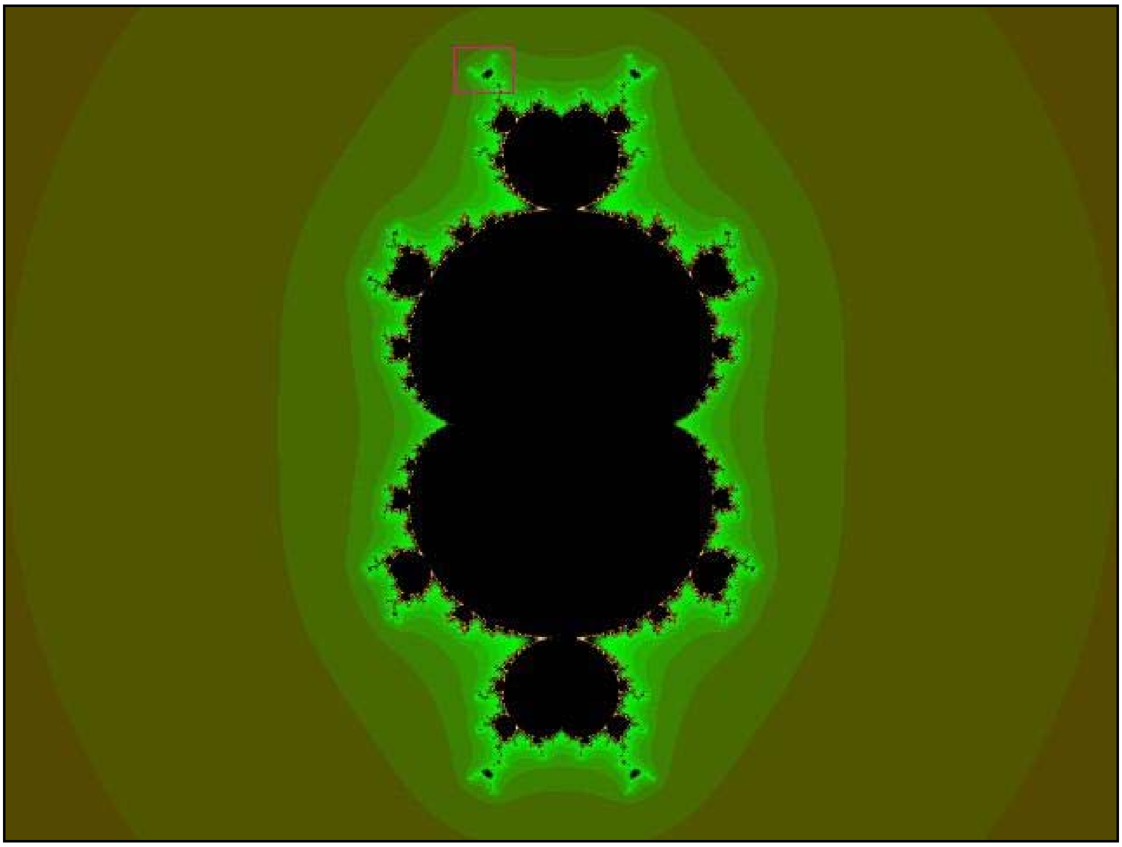

Fig 1. Mystery Start.

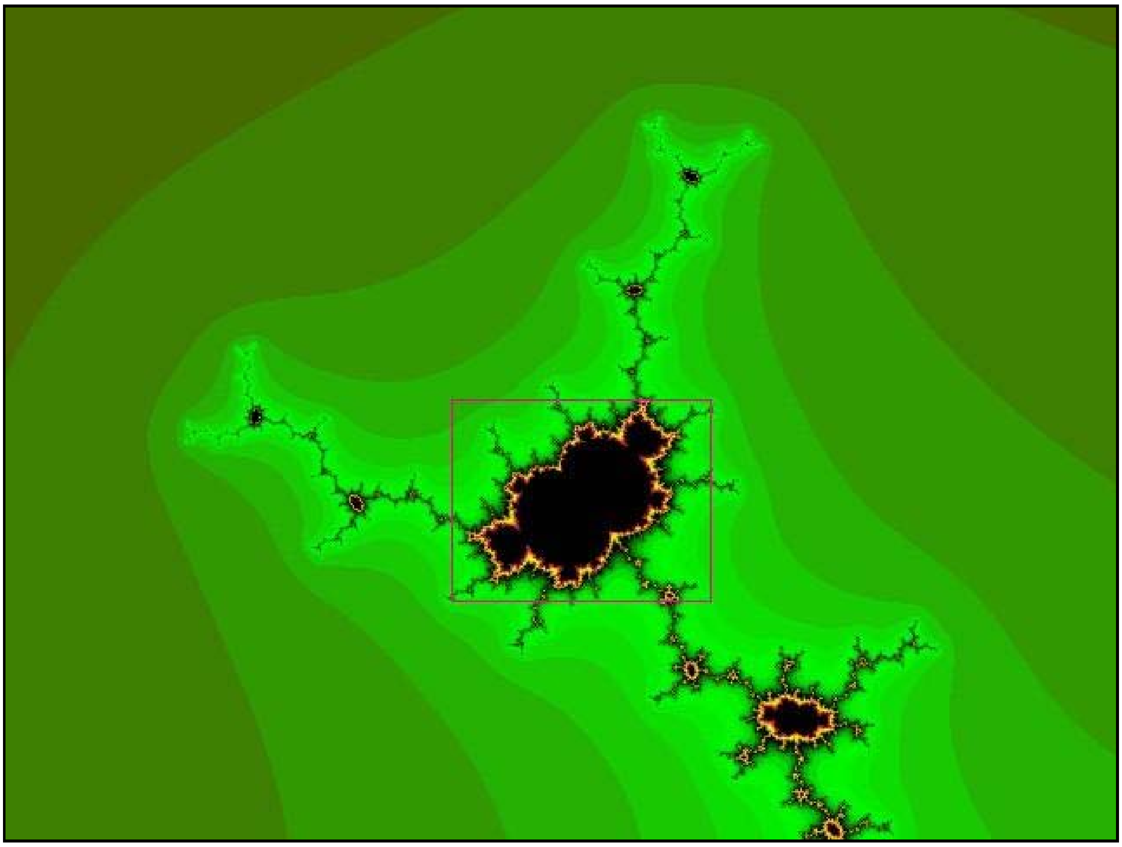

Fig 2. Mystery Zoom 1.

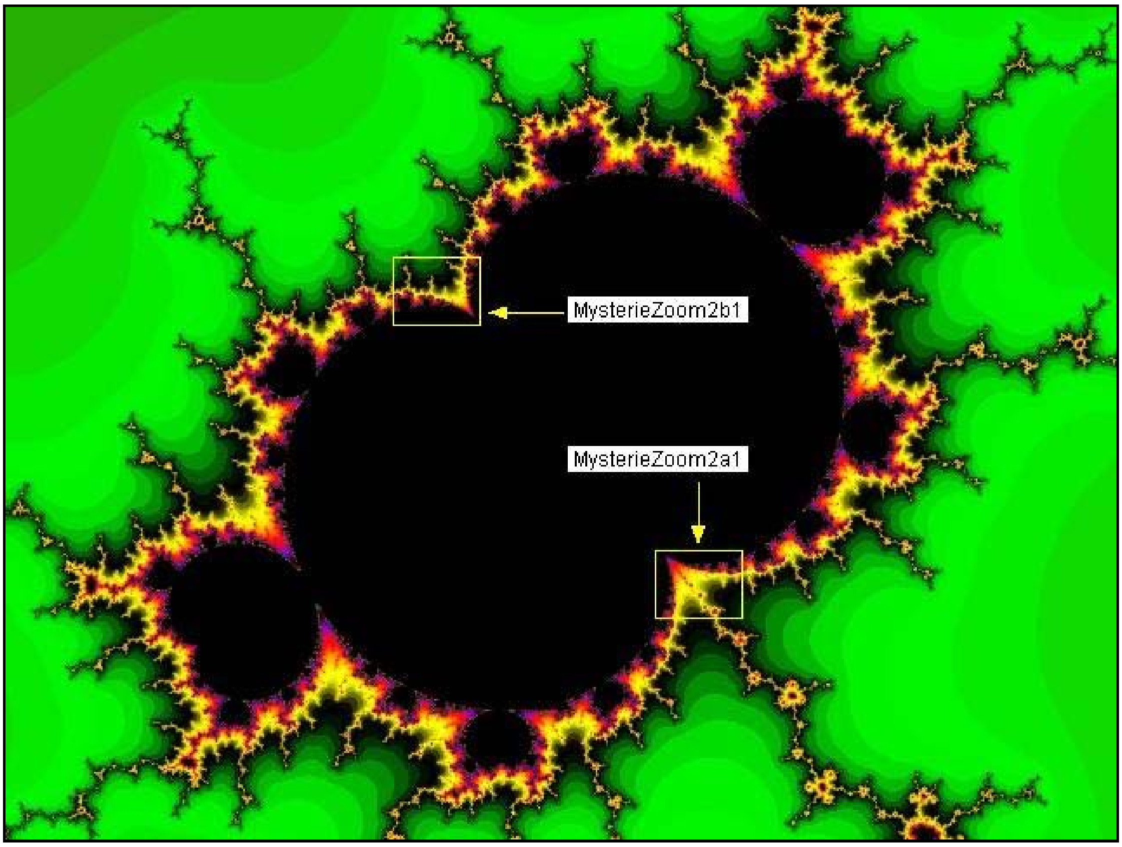

Fig 3. Mystery Zoom 2.

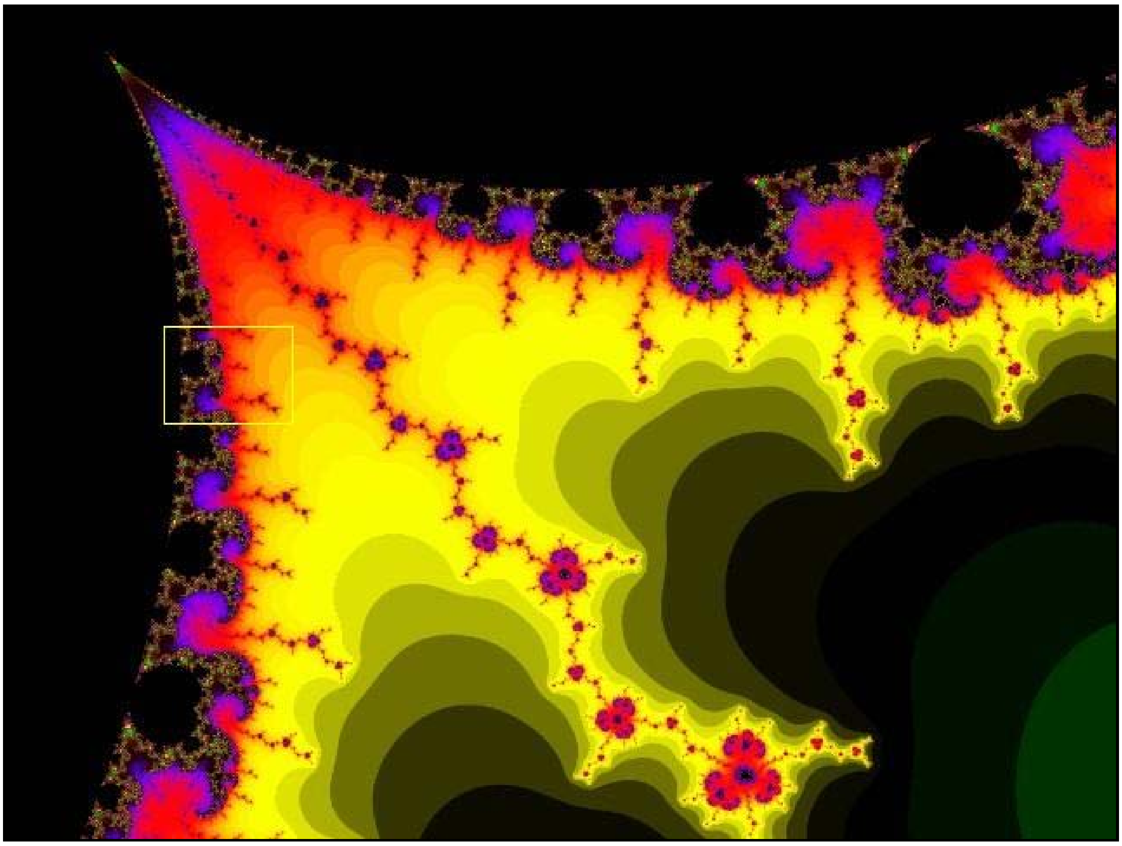

Fig 4. Mystery Zoom 2a1.

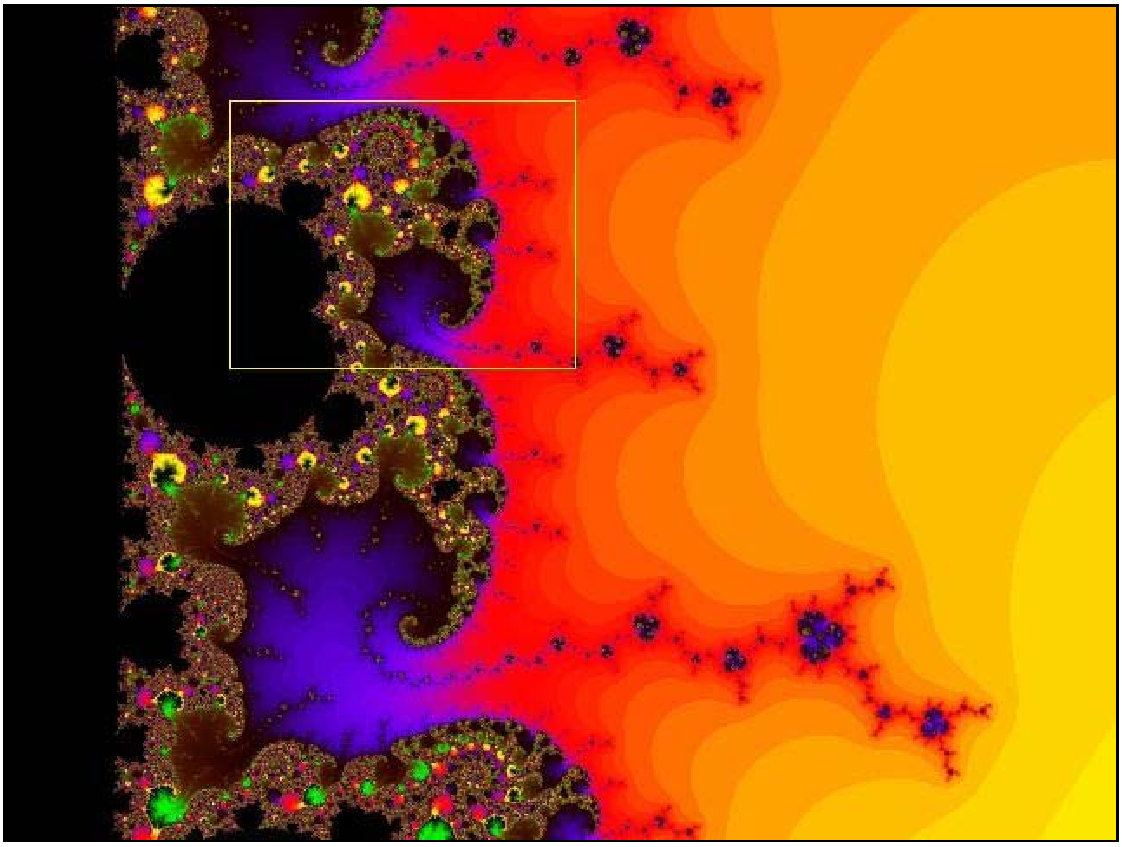

Fig 5. Mystery Zoom 2a2.

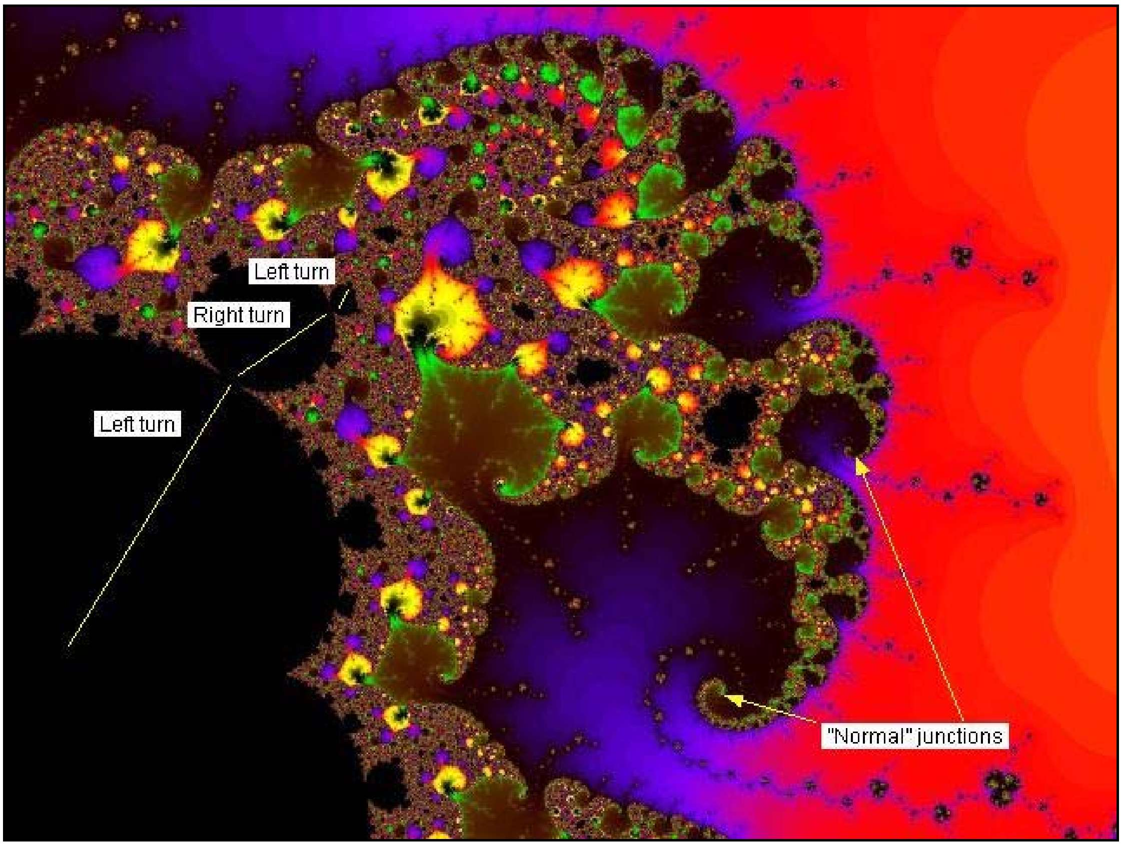

Fig 6. Mystery Zoom 2a3.

shown in fig 2, ”Mystery Zoom 1”. From the main set a filament joins the lower aisle of the mini copy. In the upper aisle however, no filament joins. Instead the two outgoing ”main” filaments leave from the upper sides of the horns (as cubic minibrots have three main filaments). From figure 3, ”Mystery Zoom 2” we perform two short zoom-sequences. In ”Mystery Zoom 2a1-3” we zoom into the lower Elephant Valley and we see that the secondary decorations join the Elephant trunks as they do in the Elephant Valley in the ordinary M set. The ”yellow-line illustrations” in ”Mystery Zoom 2a3” will be commented further down.

Now let’s have a look at the upper Elephant Valley (figures 7 - 9, Mystery Zoom 2b1-3) in order to see were the secondary decorations join the mini copy on this side. In ”Mystery Zoom 2b3” (figure 9) we see that the secondary decorations do not join the Elephants trunks. Neither they join in the top of any hierarchy of ”extra heads” as in the body side of the Seahorse Valleys (both in the quadratic and cubic Mandelbrot sets). In fact there are no extra heads in the Elephant Valley. Instead, and hear is what I call **”the Great Cubic Mandelbrot Mystery”**, ***they join in completely undefined spots.*** Further zooms do not reveal this mystery. The secondary decorations seem to join in a gap. This is very mysterious since, according what is stated above, the generic Mandelbrot set for a polynomial of any degree is connected! In the next article we shall approach this big mystery by fixing the coordinate of the right market mysterious junction and have a look at this spot from other dimensions, using the technique described in Article 22. Maybe the diligent reader is able to perform this already now before next article is published :-)

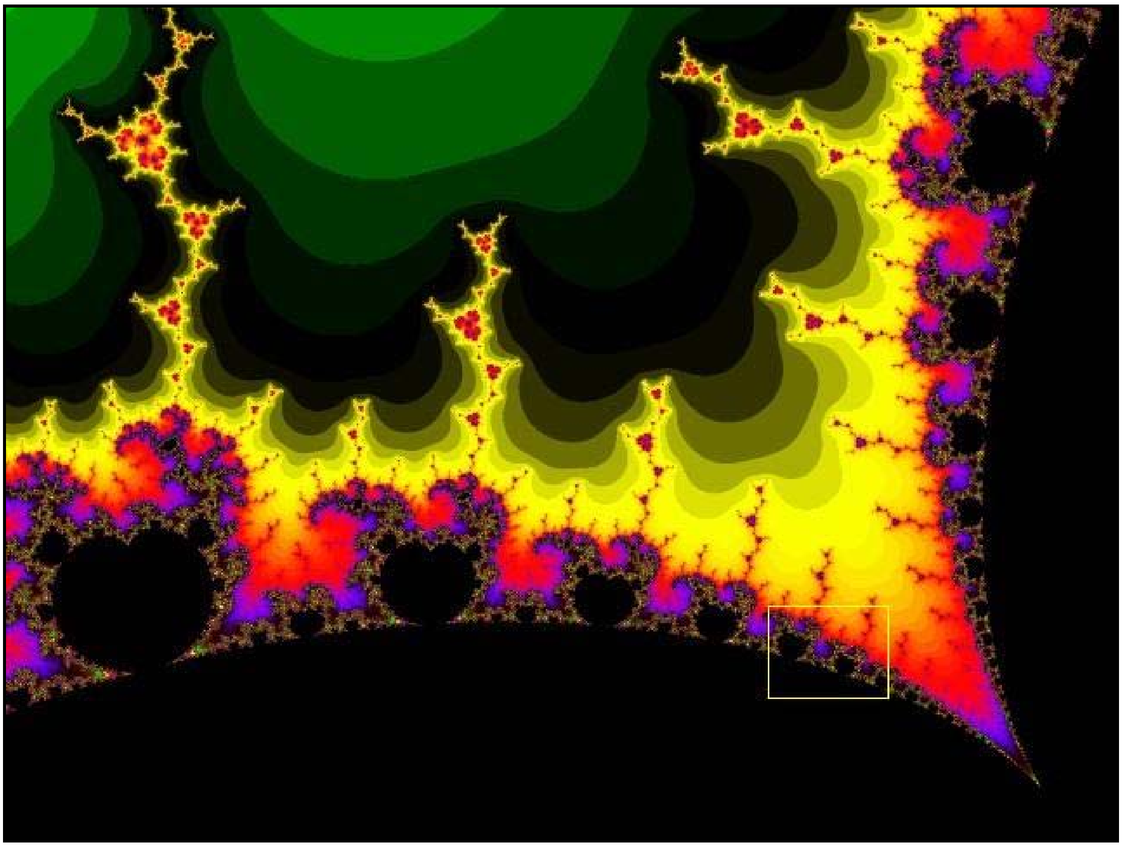

Fig 7. Mystery Zoom 2b1.

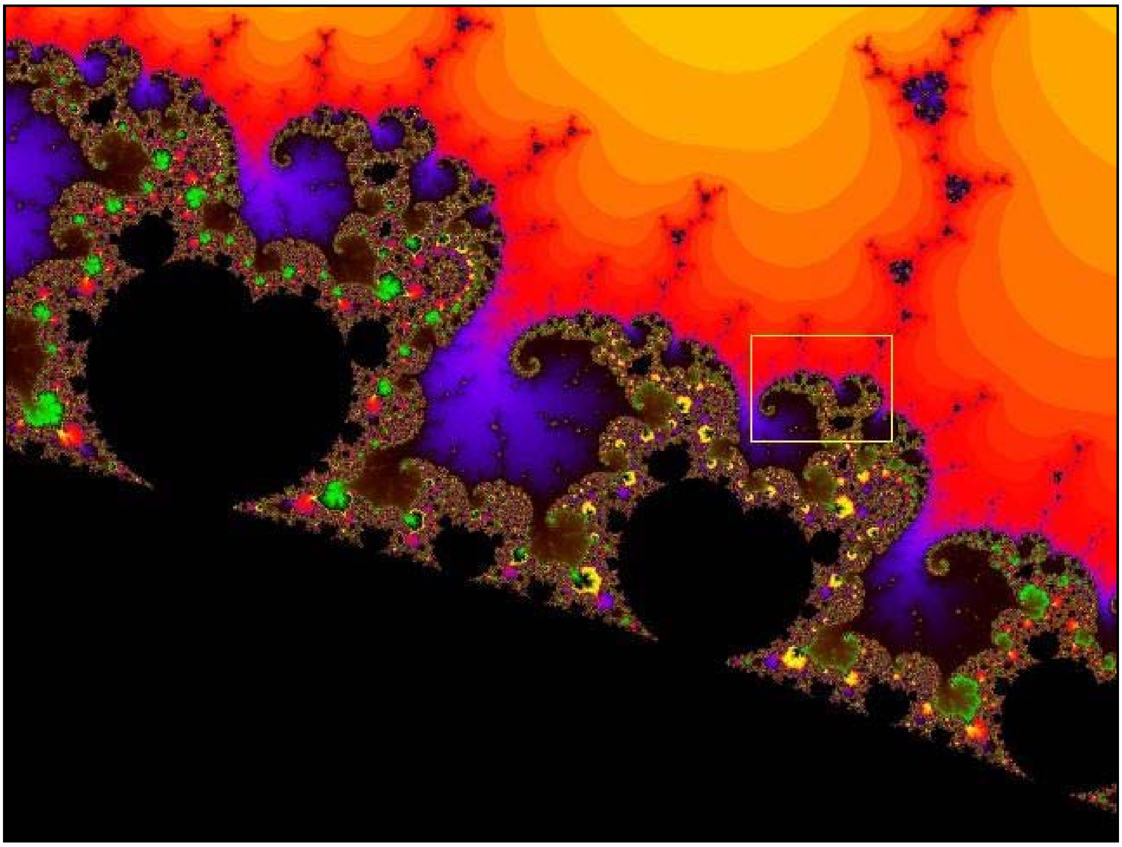

Fig 8. Mystery Zoom 2b2.

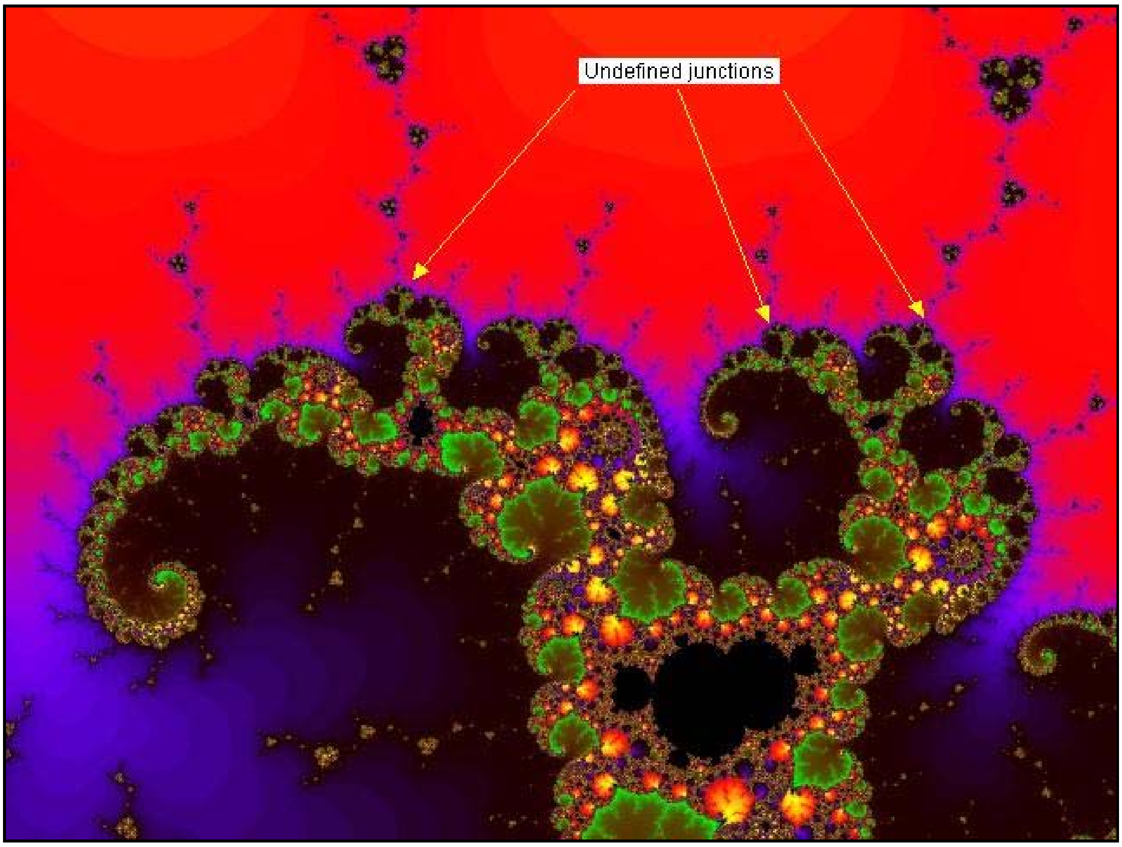

Fig 9. Mystery Zoom 2b3.

Finally we have **”the Little Cubic Mandelbrot Mystery”.** Let's return to figure 6, ”Mystery Zoom 2a3”. Like in the ordinary M set, you have to cross an infinite number of smaller and smaller buds in order to reach the filaments. That's no problem in the ordinary M set, were the buds are circles. In the cubic Mandelbrot set, however, the buds are cardeoids. That means you have to alter between left and right turns to reach the visible filament. Here is a very good ”zen buddhistic koan” to meditate upon. ***Do the appointed filament join on the left or right part of the ”last” cardeoid-shaped bud?*** I look forward to see the solution of this koan from the enlightened reader :-)

Don’t forget my ”Cubic Tutorial” and ”Pictures from Cubic Parameterspace” reachable from my index page.

Regards

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