27) Compasses

(Parameter "spaces" (= planes) of the form : $p(z) = z^d - da^{d-1}z$)

      Let's have a polynomial in the form $p(z) = z^d + kz$, $d \geq 2$. To receive the critical points, let's put the derivative to zero:

$p'(z) = dz^{d-1} + k = 0$

$z^{d-1} = -k/d$

$z = (-k/d)^{1/d-1}$ (read (d-1) - rot of -k/d).

      We only take the positive solution into account (if "d" is odd there is also a negative solution). Let's call the positive critical point "a", rewrite the parameter "k" in the form of "a" and then rewrite the polynomials;

$a = (-k/d)^{1/d-1}$

$a^{d-1} = -k/d$

$k = -da^{d-1}$

      that is our polynomials become $p(z) = z^d - da^{d-1}z$. Now let's draw the a-section for d = 2, 3, 4, and 5. The pictures are generated by the sub-module "Compasses" in my ik-module. The interior coloring is achieved by "Diff-Bailout and denote 1-periodic components (NOTE; in the following images the magnification and in the case d = 2, the center-coordinate is adjusted in order to come up with nice images).
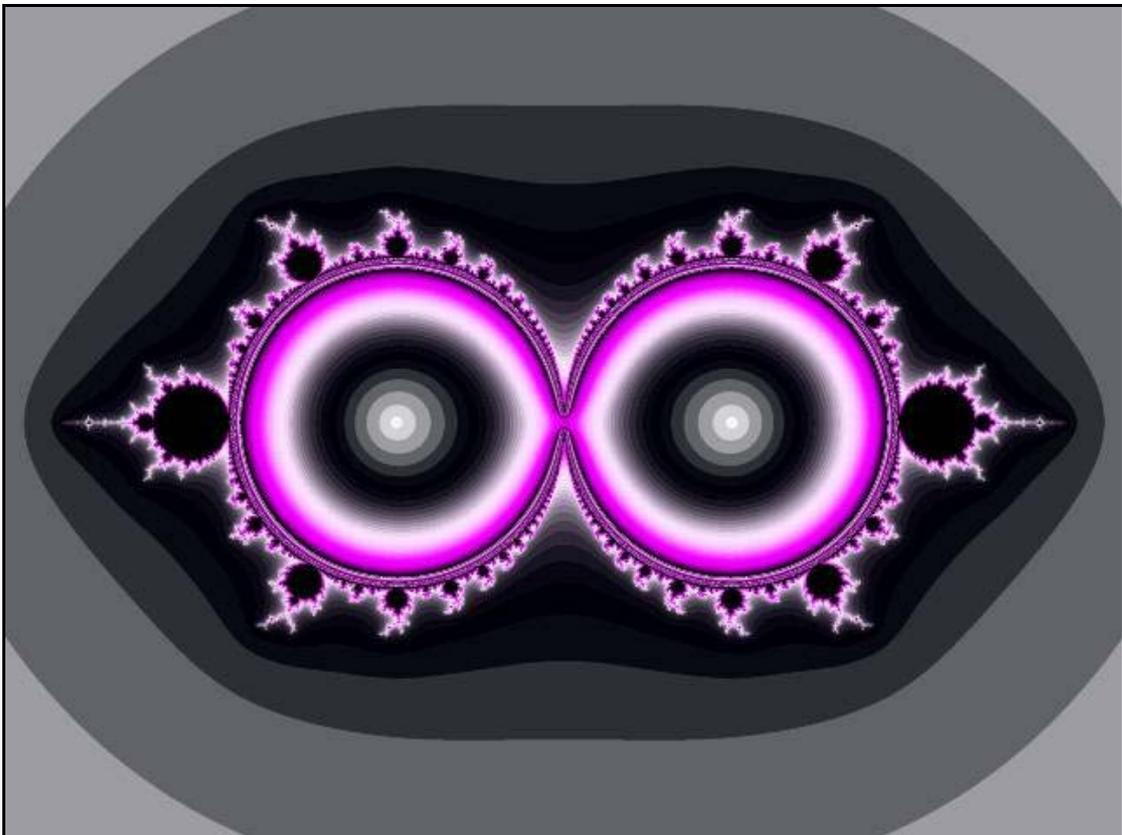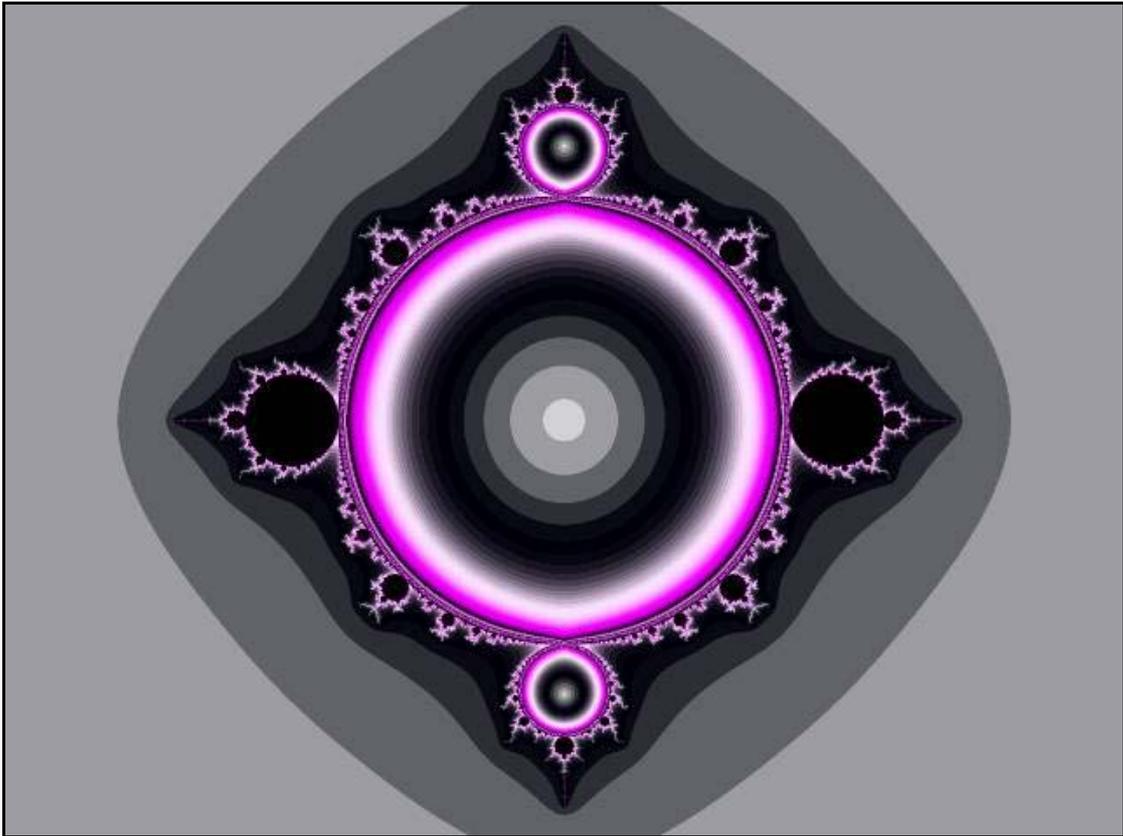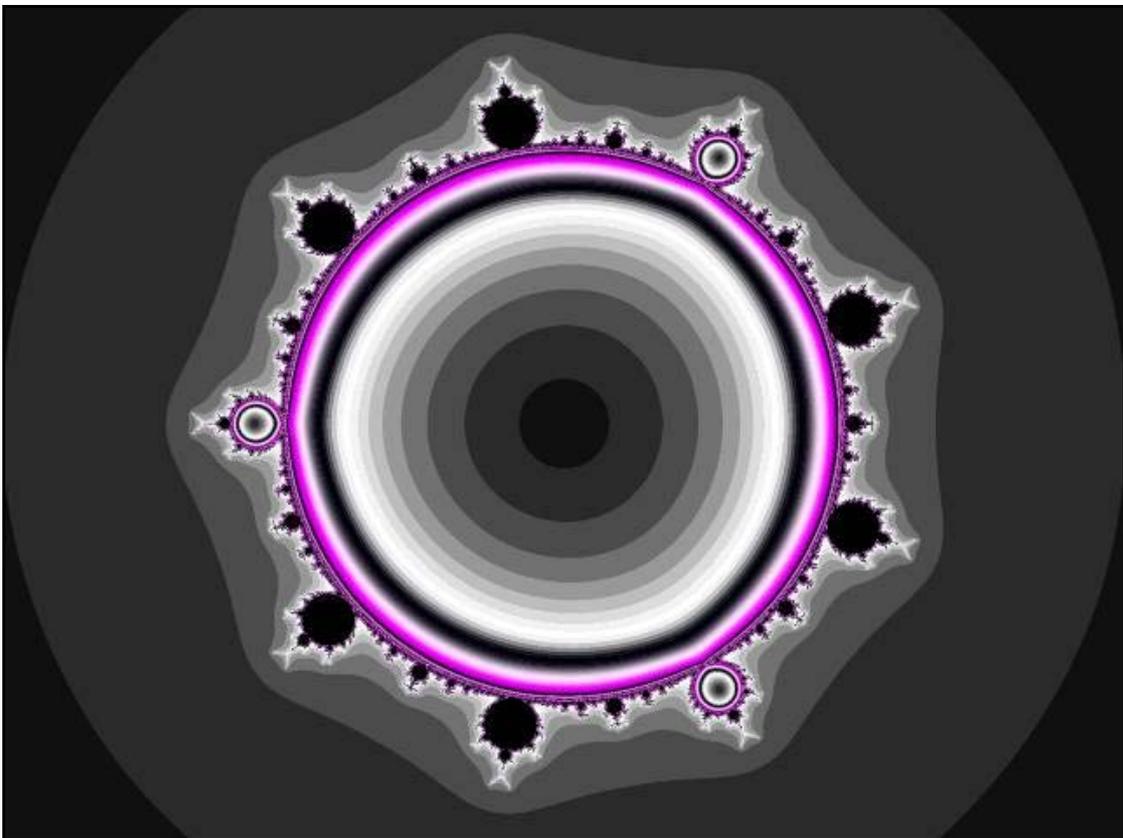

Fig 1. d = 2.

Fig 2. d = 3.

Fig 3. d = 4.

1) d = 2. We receive p(z) = z^2 - 2az, in fact two joined Mandelbrot sets with the same shape as the so called "Mandellamnda" (generated by z -> az(1 - z)). If we add a fixed parameter "b", that is, we iterate z -> z^2 - 2az + b, there are always two Mandelbrot sets at different locations, often separated from each other. If we put our quadratic polynomials in the above form and plot b-sections for different fixed "a", we always receive one exemplar of the normal Mandelbrot set (at different locations). Among others this shows that one parameter is enough to specify a quadratic polynomial. That's the reason why we put our quadratic polynomials in the well known form p(z) = z^2 + c with the critical point z = 0, when we draw the quadratic parameter plane. Generally speaking, when we study polynomials we put them **a) monic,** the coefficient before the highest term is one, and **b) centered,** the coefficient before the next highest term is zero.

2) d = 3. This gives rise to a special case of the well known formula for drawing the cubic parameter space, p(z) = z^3 - 3a^2z + b with the critical points z = +-a and "b" fixed to zero. The sets M+ and M-, generated by the two critical orbits, coalescence in this special slice (there is no B-locus). The critical orbits don't coalescence, but they have opposite signs. The branches are solely made up of copies of the Mandelbrot set. The four heads are like the head of the Mandelbrot set. On the side of the body down the eight seahorse walleys, the number of branches of the main stars instead of 3, 5, 7, 9 etc is 4, 6, 8, 10 etc. The edges of the four spikes have the following coordinates:

Left spike: a = -1 + 0i. The orbit of the critical point z = +a is; -1 -> 2 -> 2 -> 2 etc (z = 2 is a repelling fix point). The orbit of the other critical point z = -a (for the coordinate a = -1 + 0i) is the same as the for the critical point z = +a for the coordinate a = 1 + 0i just below. On the dynamical plane this set of parameters, (a, b) = (-1+0i, 0+0i) and (a, b) = (1+0i, 0+0i) gives rise to a non fractal Julia set in the form of a straight connected line from -2 + 0i to 2 + 0i. That is exactly the same shape as the Julia set generated by c = -2 (the very tip of the spike of the Mandelbrot set) for quadratic polynomials.

Right spike: a = 1 + 0i. The orbit of the critical point z = +a is; 1 -> -2 -> -2 -> -2 etc. Gives rise to the same Julia set as the above.

Upper spike: a = 0 + i. The orbit of the critical point z = +a is; i -> 2i -> -2i -> 2i -> -2i etc (2i -> -2i is a repelling 2-periodic cycle). Gives also rise to a 4 units straight Julia set, but this time a vertical one, from 0 - 2i up to 0 + 2i.

Down spike: a = 0 - i. The orbit of the critical point z = +a is; -i -> -2i -> 2i -> -2i ->2i etc. The Julia set is the same as for upper spike.

3) d = 4. This compass may occur in a slice of the six dimensional parameter space of a certain parametrizing of fourth degree polynomials. However this compass does not occur in the standard-parametrizing for quartics described in the next article. Instead of 4 heads (for d = 3) there are now 9 heads (three of them are 1-periodic). The main branch of these heads are not spikes (can be considered as 2-stars), but have 3-stars.

4) d = 5 and higher. I left to the diligent reader to see what happens when "d" increases (just change the value of "d" under parameter "exponent").
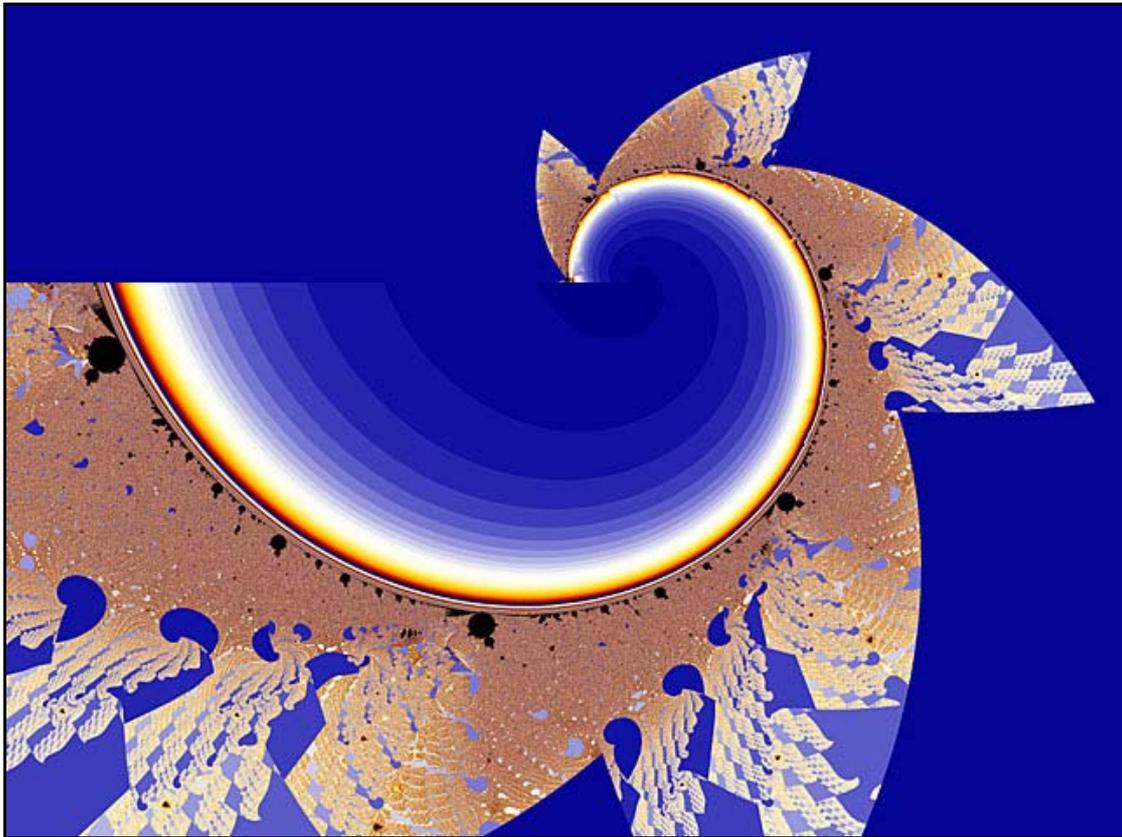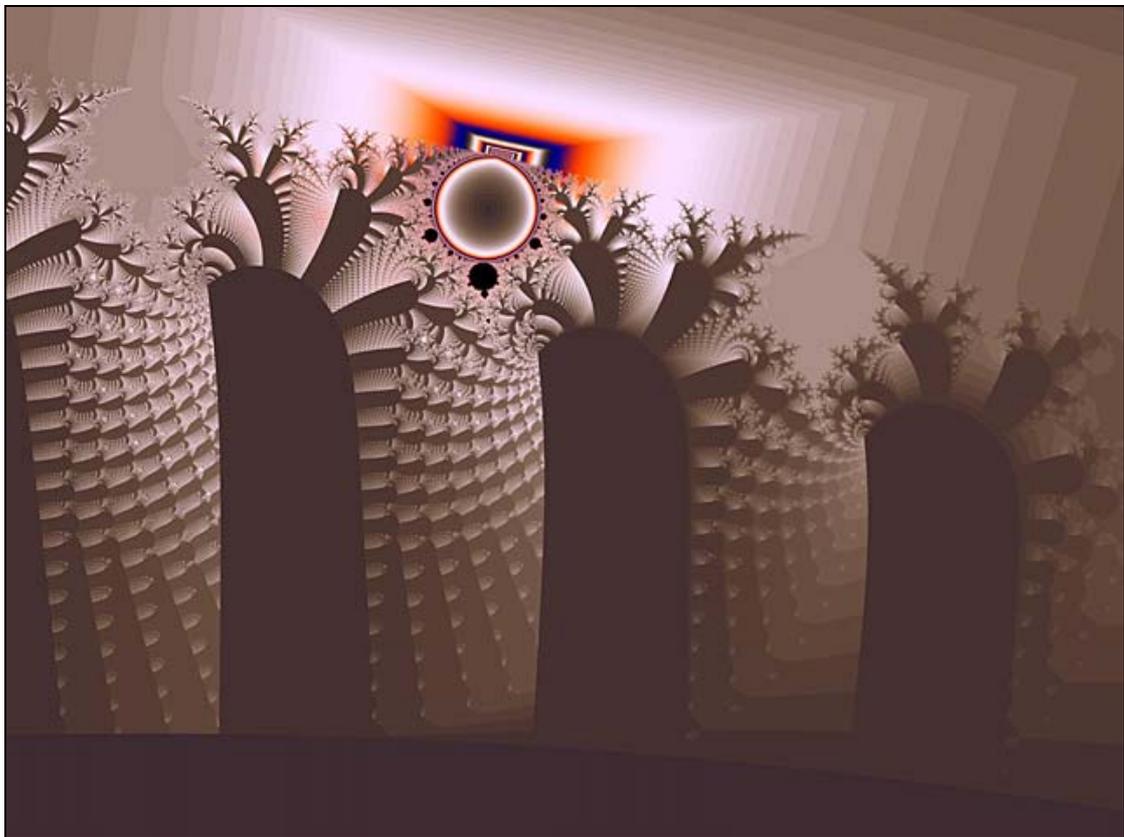
Fig 4. d = -4.54545354 + 1.875778i.

Fig 5. d = 11 + i. Detail.

The compass-modul in UF also allows you to choice not only integers för "d", but any real and complex number. The critical "z" will always initiated as first input. This means, whatever the forms will be (often broken and bizarre patterns), you will always find copies of the ordinary Mandelbrot set if you zoom in. Hope figures 4 and 5 will be appetizers. Play and enjoy!

Regards

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