

29) Four-dimensional quadratic parameter space

This is a little bit of fiction

Being a diligent reader after having read Article 28 and meditated upon the Mega-troll formula:

$$(1) p'(z) = d(z - a_1)(z - a_2) \dots (z - a_{d-1})$$

$$(2) a_1 + a_2 + \dots + a_{d-1} = 0.$$

(3) a_1, a_2, \dots, a_{d-1} are the critical points,

and it's consequences for quadratics, where we obtain:

$$p'(z) = 2(z - a_{2-1}) = 2(z - a_1) = 2z - 2a_1$$

and obtain using condition (2)

$p'(z) = 2z$ (the sum of the critical point is put to zero, the only critical point being a_1) and obtain after integration:

$$p(z) = z^2 + b$$

which is the well known iteration formula $z \rightarrow z^2 + b$, the Mandelbrot set constituted of those parameters "b" for which the critical point $z = 0$ has a bounded orbit, I made the following reflection:

The condition (2) says that the sum of the critical points shall be zero (in order to make the polynomial centered). Well, if we skip that condition we would obtain: $p'(z) = 2(z - a) = 2z - 2a$ (there is only one critical point, so we say "a" instead of "a₁").

Now we obtain after integration:

$$p(z) = z^2 - 2az + b$$

In this case, like in cubics, we have two complex parameters, (a, b), making the parameter space to a four-dimensional hyper-space. So I wrote a sub-module making it possible to draw all 6 perpendicular 2D-slices with possibilities to rotate between the planes. Of course Switch mode is included. So let's play with it.

However tell that happiness that last (Swedish tale). Now my master demonstrated for me: Look here. Let's draw the $(b_{\text{real}}, b_{\text{imag}})$ gliding along the $(a_{\text{real}}, a_{\text{imag}})$ - axis'. The default very standard Mandelbrot set only change position as you type different values in the boxes for a_{real} and a_{imag} . What's the meaning of that? And if we instead draw the $(a_{\text{real}}, a_{\text{imag}})$ gliding along the $(b_{\text{real}}, b_{\text{imag}})$ - axis' we obtain two mostly separated M sets. In default we obtain a "Mandel-lambda" but with other coordinates along the horizontal axis.

Oh, yeah, in the default $(a_{\text{real}}, b_{\text{real}})$ shows up with a non-fractal horseshoe. However these are stapled Mandelbrots seen from the side.

Now let's play with Switch mode (see Article6). There is no Julia set obtained from any of the plotted planes which we can not obtain from the standard parameter plane where the ordinary quadratic M set reside. Oh yeah, the locations can differ, but their shape are the same. In fact my friend, the

ordinary M set, residing on the parameter-plane (b-plane) when iterating $z \rightarrow z^2 + b$, initializing "z" to zero, contains all dynamic information for quadratic polynomials."

The dear readers are now invited to check this by drawing a Julia from this module, and I will try to draw a similar from the ordinary Standard (Mandelbrot) module. Please do not use heavy filters, it's the forms that are asked for.

Regards

Ingvar

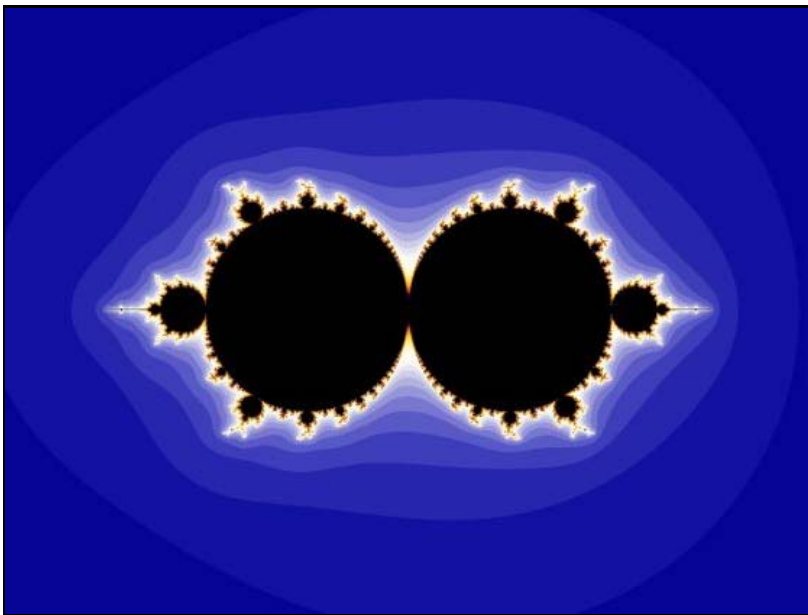


Fig 1. a_{real} , a_{imag} .

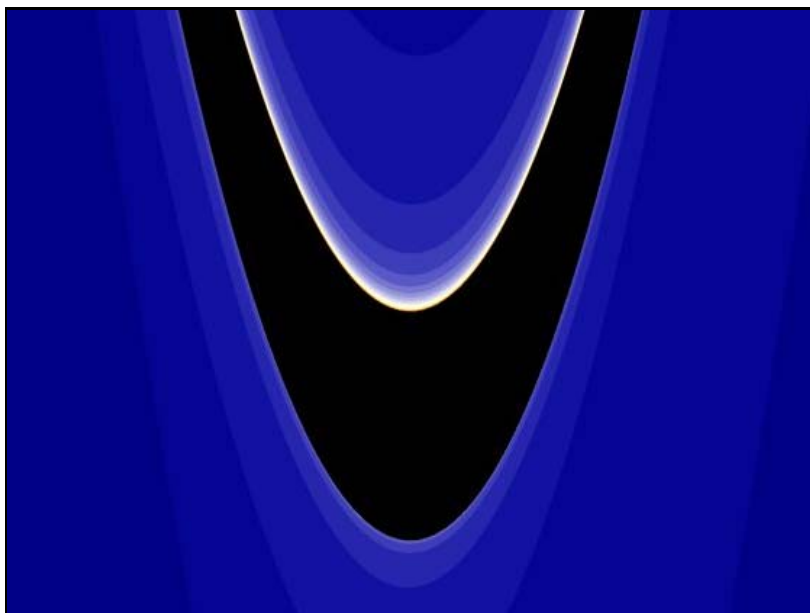


Fig 2. a_{real} , b_{real} .

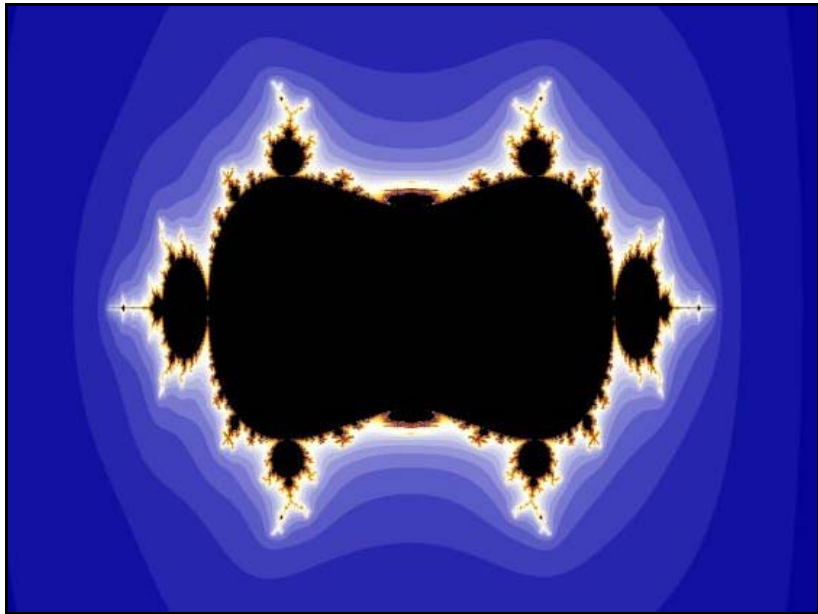


Fig 3. $a_{\text{real}}, b_{\text{imag}}$.

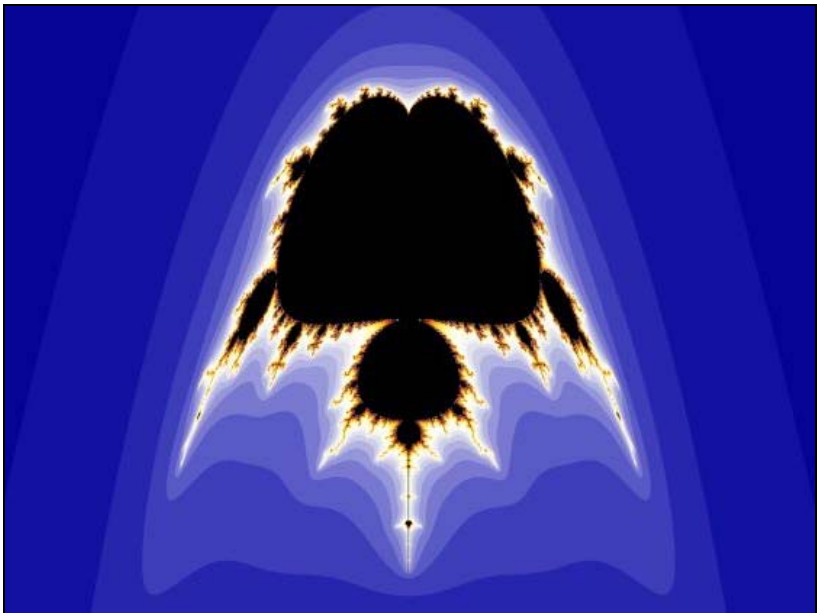


Fig 4. $a_{\text{imag}}, b_{\text{real}}$.

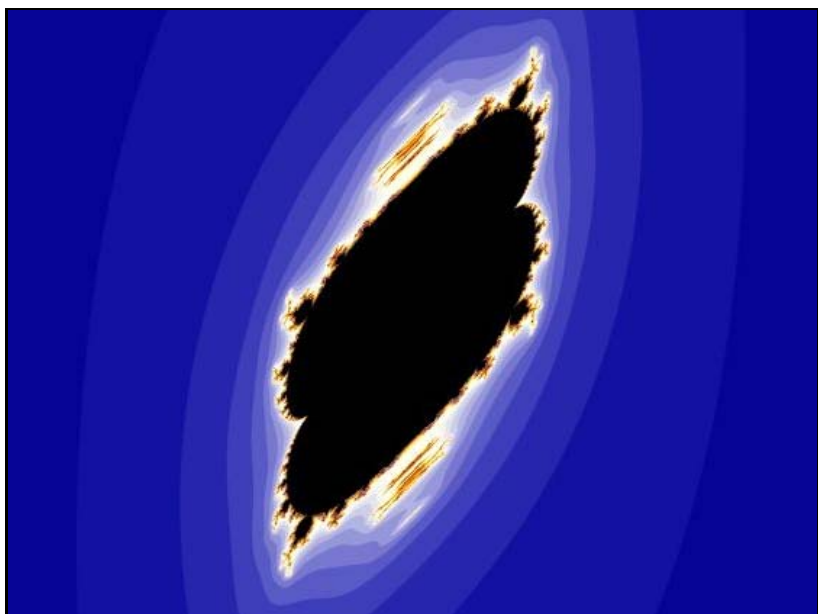


Fig 5. $a_{\text{imag}}, b_{\text{imag}}$.

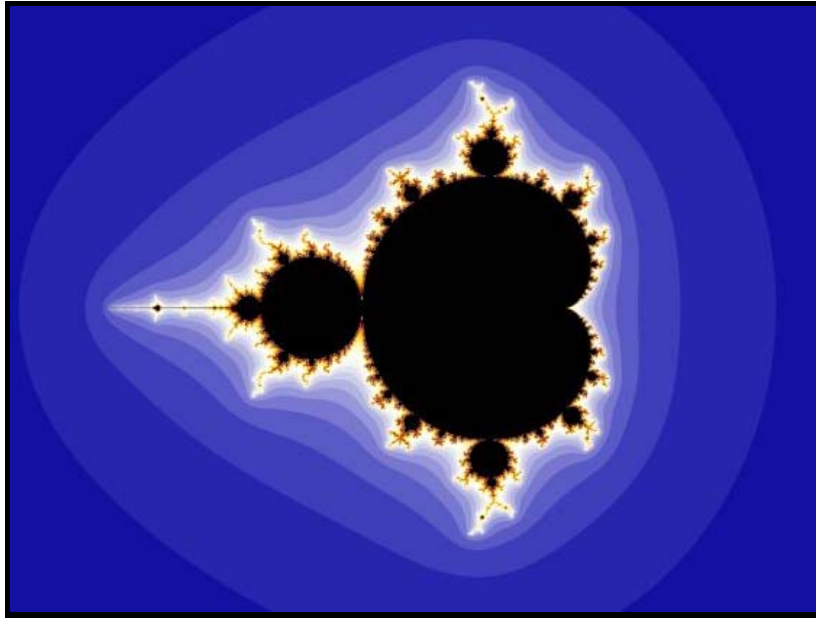


Fig 6. b_{real} , b_{imag} .