32) Yet more about Quartics with a first attempt to Pentics

Now let’s go further with some speculations. In ”Confusion1HeadJulia” the connected components obviously have their shapes of ”CubicHeadJulia”. This is quite logical if we consider from where we have picked up the seed for this Julia set. How about the other cases of quartic Julia sets where one critical point escapes and the other two critical points have bounded orbits but don't coalesces, as in figure 1, ”QJ3” borrowed from Article 30? Do these connected components, where one of these is containing the two bounded critical points, have the form of cubic connected Julia sets? My theory is yes. Look at figure 2, ”A connected Cubic Julia”. This is a connected cubic Julia set as both critical points belong to the ”filled-in” Julia set. This connected cubic Julia set have almost the same shape as the connected regions in the quartic Julia set QJ3 where also two critical points belong to the filled-in Julia set, the third critical point being outside. As the seed for this Julia is NOT picked from a component with the shape of the generalized cubic Mandelbrot set (in which case two critical point would have coalesced) the natural question arise. Is this seed picked from a component that is situated in another slice of cubic connectedness locus? That is the same as asking the question: Are there 4D-slices of the six-dimensional parameter space of quartic polynomials that contains elements which are copies of the four-dimensional cubic connectednes locus? I will say probably yes. Below the theory of me:

In cubic parameter space: Parts of 3D-slices of cubic parameter space for which one of the two critical points has a bounded orbit are made up of towers whose 2D-cuts are constituted by the quadratic connectedness locus, that is the Mandelbrot set (see ”Towers”). Parameter-seeds from these ”Mandy-towers” give rises to disconnected Julia sets with enclosed regions. These enclosed regions have the shape of quadratic connected Julia sets. On the other hand there are also copies of the Mandelbrot set in 2D-slices of cubic connectedness locus, the locus for which both critical points have bounded orbits. Parameter-seeds from this give rise to connected Julia sets where some (connected) components have the forms of quadratic Julia sets.

In the same way in quartic parameter space: Now in the same way parts of 5D-slices of quartic parameter space for which two of the three critical points have bounded orbits are made up of ”5D- towers” whose ”4D-cuts” are constituted by the cubic connectedness locus. Parameter- seeds from these ”ccl-towers” give rise to disconnected Julia sets with enclosed regions. These enclosed regions have the shape of cubic connected Julia sets. On the other hand there are also copies of the ccl in 4D-slices in quartic connectedness locus, the locus for which all three critical points have bounded orbits. Parameter-seeds from these give rise to connected Julia sets where some (connected) components have the forms of cubic Julia sets.

The reasons for the above theory are: 1) The appearance of components in some quartic Julia sets having the shape of connected cubic Julia sets in which one of these components contains two of the three critical points. These two critical points may or may not coalesce. 2) The existence of

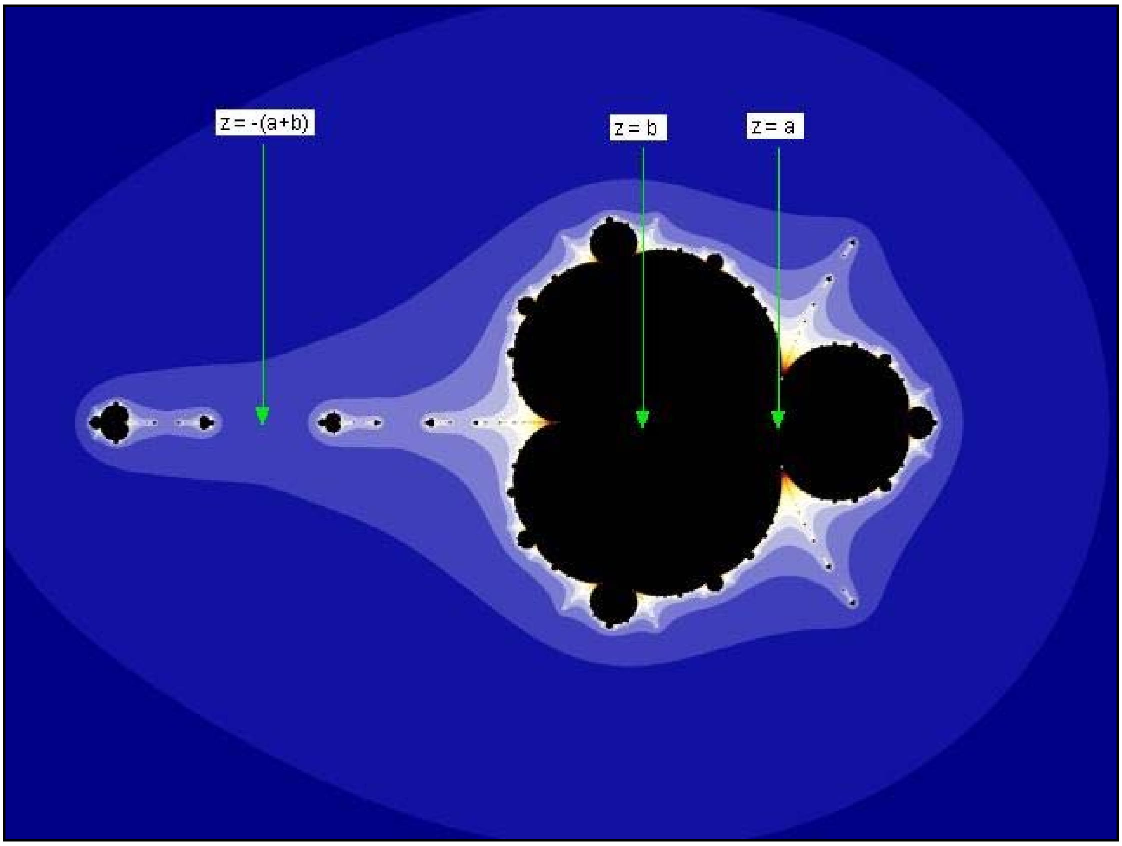

Fig 1. The disconnected Quartic Julia QJ3 from Article 30.

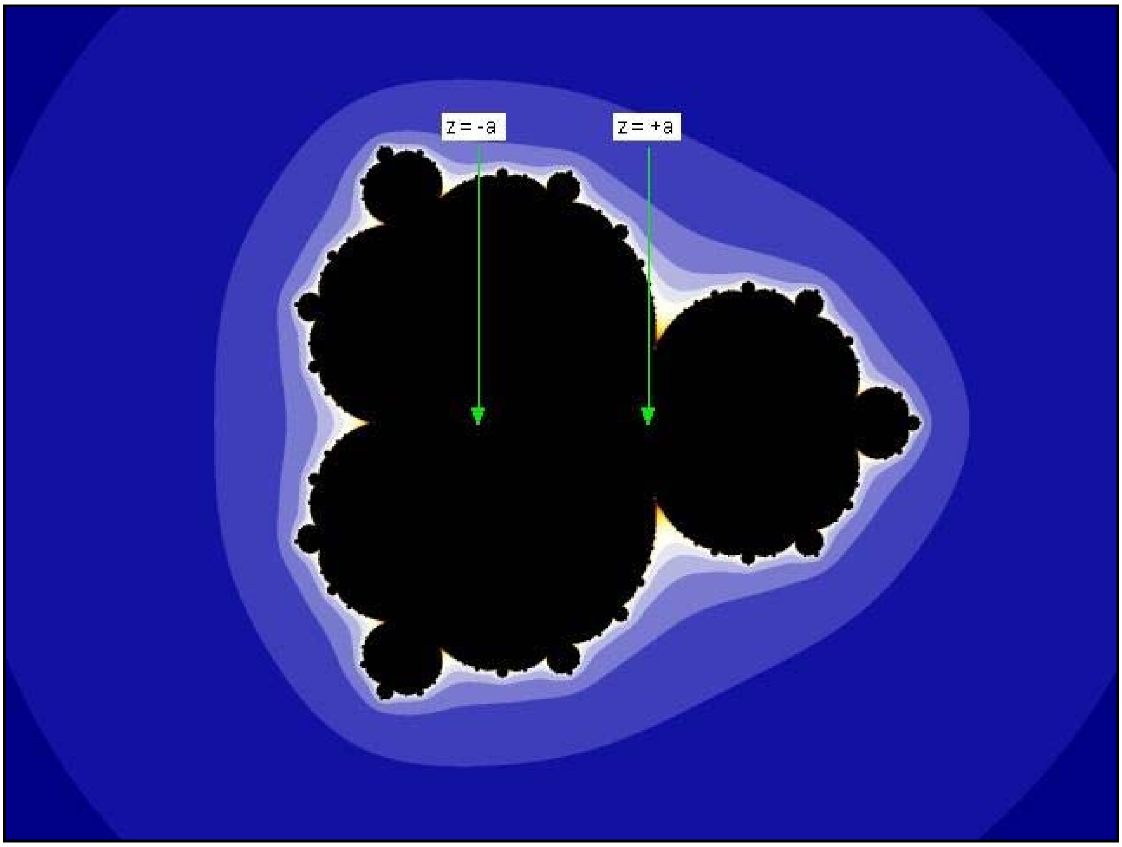

Fig 2. A connected Cubic Julia.

Fig 3. Thorns.

copies of the generalized cubic Mandelbrot set in 2D-slices may indicate that copies of the full four-dimensional cubic connectedness locus also may appear in 4D-slices of the six-dimensional quartic connectedness locus.3) The third reason is an old ”rotation-exercise” performed in the summer 2001 by me. This rotation is performed in a similar way as described in Article 22. The results are described below.

Accomplishing this article is a second parameter set called ”Views” (I’ve not posted the JPEG’s of these since it would be about an 800kb posting). Here I've fixed the center of the cubic Mandy in Confusion2 (oh yeah, I've made another version using ”QCL SetBorders” in a single layer). Thereafter rotations are made around this spot in 9 of the perpendicular slices (which was there in the old quartic module of Stig) of the six-dimensional hyper space of quartic polynomials in a like manner we have done in Article 22 were we did a similar rotation in the four-dimensional hyper space of cubic polynomials. What we find is that the environment around this six-dimensional coordinate is almost the same in most of the perpendicular slices. In some of the cases the structures are very very stretched. However in some others the motives are almost identical with the "start-view" (c_{real}, c_{imag}) although their unmagnified parent fractals are quite different. These motives may indicate that cubic minibrots also occur in the other perpendicular systems of slices at certain fixed values. However if you investigate some of the cubic minibrots in these other slices you will find that they are defect in their details. Moreover they are defect in the same way as the prototype cubic Mandelbrot set would be defect if you would move a very short distance along one of the a-axis’ (a_{real} or a_{imag}) or perform a very little rotation. This is one of the reasons for the

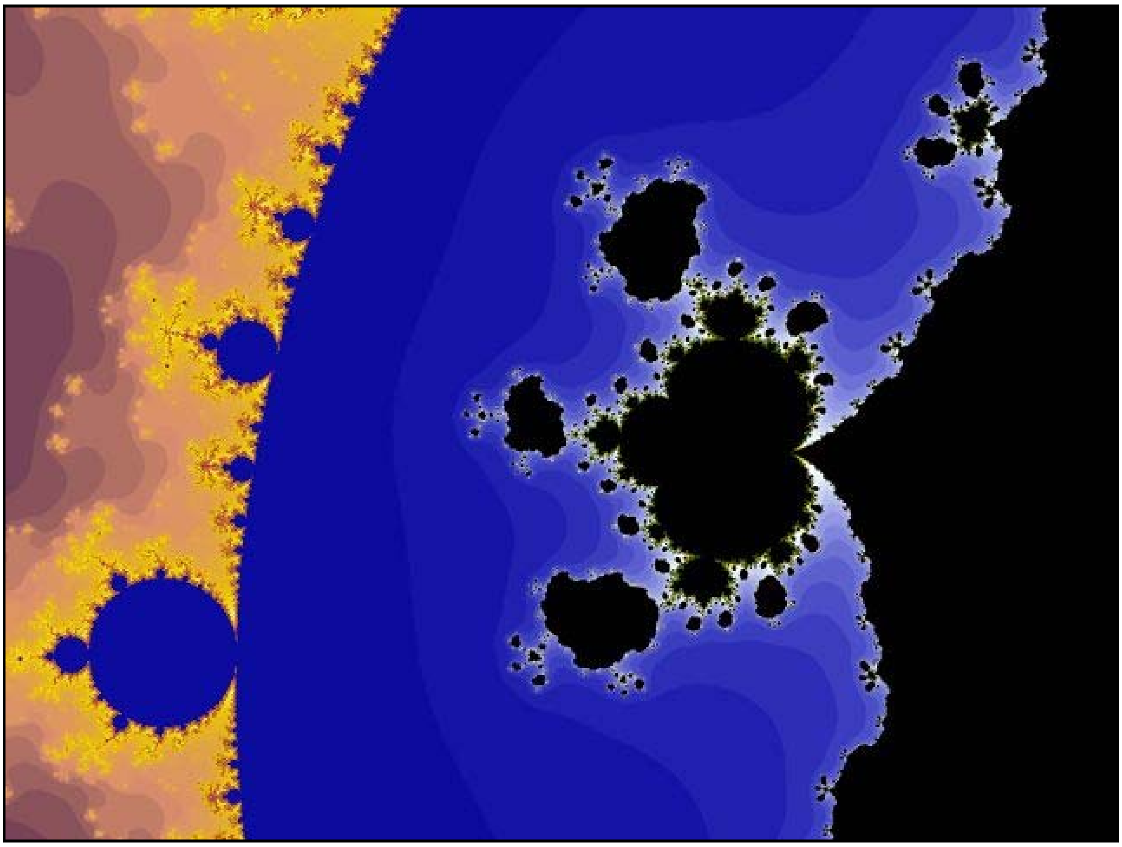

Fig 4. PenticsTempDemo 1.

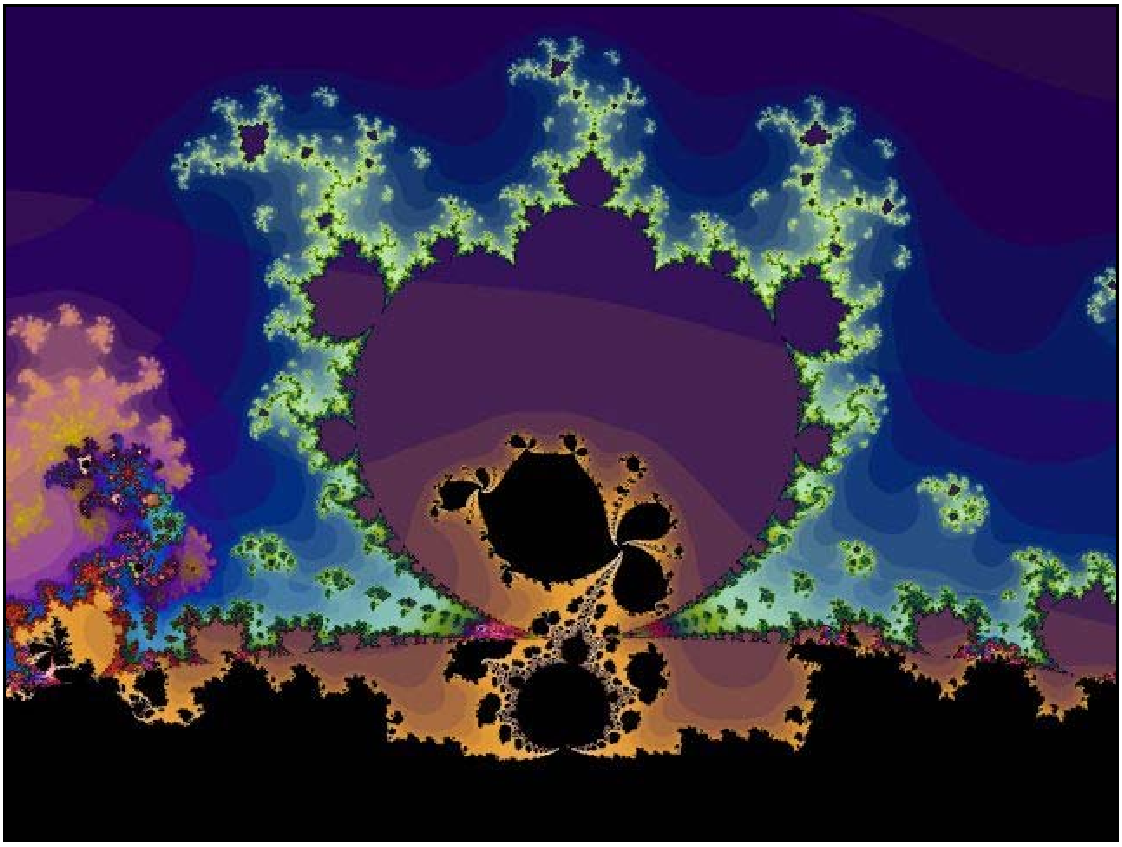

Fig 5. PenticsTempDemo 2.

above theory about the appearance of the full four-dimensional ccl in quartic parameter space and maybe yet higher parameter spaces.

Where do we find cubic minibrots inside subsets also having the shape of the cubic Mandelbrot set? Well they seem not to be found in quartics. However I will make a prophecy that this phenomena would be found in some slices of the eight-dimensional pentic parameter space. Pentic polynomials have four critical orbits that give rise to four sets in the parameter space. The phenomena with cubic minibrots iniside subsets also having the shape of the cubic Mandelbrot set according to my prophecy will occur when two subsets coalesces forming the outer subset and the other two subsets coalesces forming the inner subset. In an analogous manner in slices of pentics where three of the four critical points coalesce, the three resulting subsets would contain copies of the generalized quartic Mandelbrot set ($z \to z^4 + c$). When these subsets are inside the fourth subset, quartic Julia-like components are attached to the acupuncture points. Further more the border of the fourth subset has the shape of the ordinary quadratic Mandelbrot set.

In order to verify this I wrote a temporary module named ”PenticTemp3” (by modifying CubicParameterspace3), included in the zip-folder for the parameter sets. It must be into the UF’s formula folder in order to work. It deals with the iteration-formula $z \to z^5 + az^4 + b$. This is a non-centered parametrization of pentics, the critical points are obtained in the following way:

$p(z) = z^5 + az^4 + b$

$p'(z) = 5z^4 + 4az^3 = 0$

$p'(z) = z^3(5z + 4a) = 0$

Case1: $z^3 = 0$ gives $z_1 = z_2 = z_3 = 0$ giving rise to the three coalescing subsets.

Case2: $5z + 4a = 0$ gives $z_4 = -4a/5 = -0.8a$ giving rise to the fourth non-coalescing subset.

Figures ”PenticsTempDemo 1” and ”PenticsTempDemo 2” are details of the b-plane when ”a” is fixed to 1.2 + 0i.

When the full optimal iteration-formula for pentics is derived according to the method described in Article 28 somewhere in the future and published by probably Stig, the above prophecy may be fully verified :-)

With this prophecy the chaotic series finally has come to an end, and that's a good thing since the articles are tending to be more and more tedious. Eventually future theoretical articles about fractals will stand by themselves.

Thanks for enduring my articles.

Regards

Ingvar