

5) The Parameter Plane and Quadratic Connectedness Locus

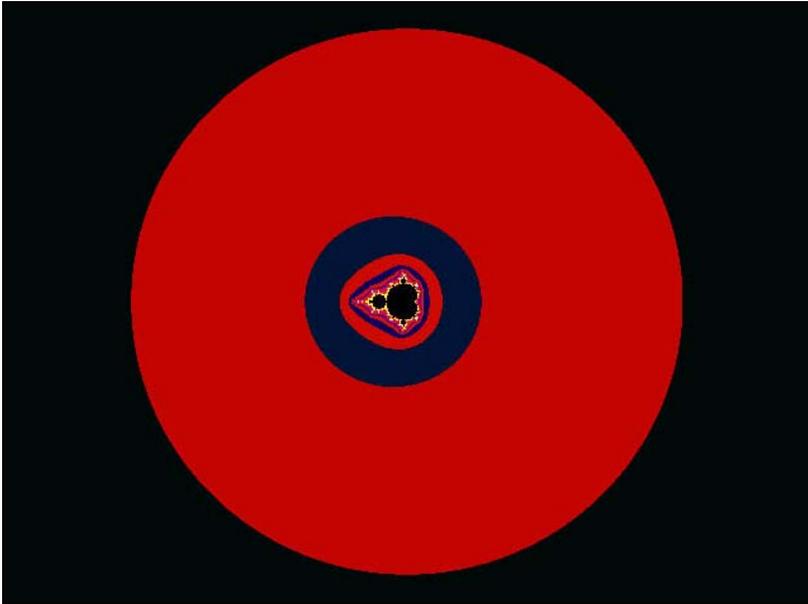


Fig 1. Adam Distant.

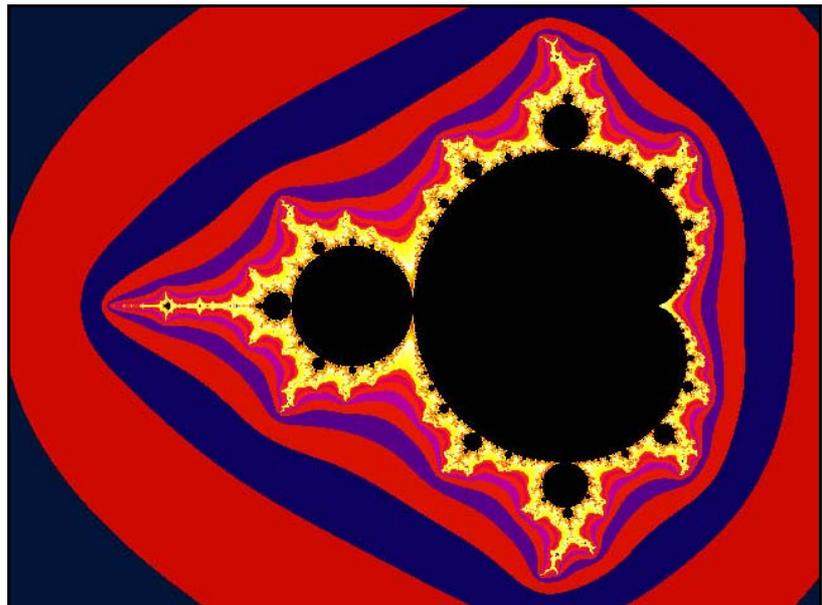


Fig 2. Adam.

The shape of the Julia set for the dynamical process $z \rightarrow z^2 + c$ is governed solely by the complex parameter "c". Since there are two main types of Julia sets for the above process it would be interesting to plot those parameters "c" which give rise to connected Julia sets. This is very simple because, as we seen earlier, if the critical point belongs to the filled-in Julia set (that's the Julia set plus eventually enclosed regions) the Julia set is connected, otherwise not. So we only for each $c = \# \text{pixel}$ have to test if the critical point $z = 0$ has a bounded orbit in which case the Julia set is connected. *It is those "c" who form that famous set which in honor of Benoit Mandelbrot, is called "the Mandelbrot set"*.

Since the Mandelbrot set is the set of those parameters that give rise to connected quadratic Julia sets, it is also called *Quadratic Connectedness Locus*. Thus it can be stated:

1. Either the Julia set for a certain c -parameter is connected, and thus the critical point $z = 0$ belongs to the filled-in Julia set, that is has an orbit that is bounded forever around a limited area (a radius of max 2) around the center of the dynamical (z) plane. In that case this c -parameter belongs to the Mandelbrot set and the computer program light the corresponding pixel ($c = \#pixel$) with in most cases black color (if you don't use any inside color-routine).

2. Or the Julia set for a certain c -parameter is disconnected, and thus the critical point ($z = 0$) does not belongs to the filled-in Julia set. In that case this c -parameter does not belongs to the Mandelbrot set and the computer program light the corresponding pixel ($c = \#pixel$) with a color corresponding to the number of iterations required to take the critical point $z = 0$ outside a certain radius. This radius must be at least 2 (bailout = 4 in UF). I for certain reason in these articles have put this radius to 10 (bailout = 100 in UF).

Figures 1 and 2 above ("Adam Distant" and "Adam") show the Mandelbrot set.

The area outside the outer circle are those parameters $c = \#pixel$ FOR WHICH the critical point $z = 0$ (on the dynamical plane) is already in the target set. That is the Julia set is such extreme Cantor dust that the target set, even for an escape radius = 10, is bent to embrace $z = 0$. This can be checked out by the Switch Mode.

The following red area are those parameters $c = \#pixel$ FOR WHICH the critical point $z = 0$ requires one iteration to reach the target set.

The following dark-blue area (also visible in the corner-regions in the second figure) are those parameters $c = \#pixel$ FOR WHICH the critical point $z = 0$ requires two iterations to reach the target set.

And so it goes on. The level sets near the Mandelbrot set are here colored in nuances of yellow-white-red. Note that the whole black central area (made up of those c -parameters FOR WHICH the critical point $z = 0$ never escapes) is the Mandelbrot set, and not only the fractal border.

Regards

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