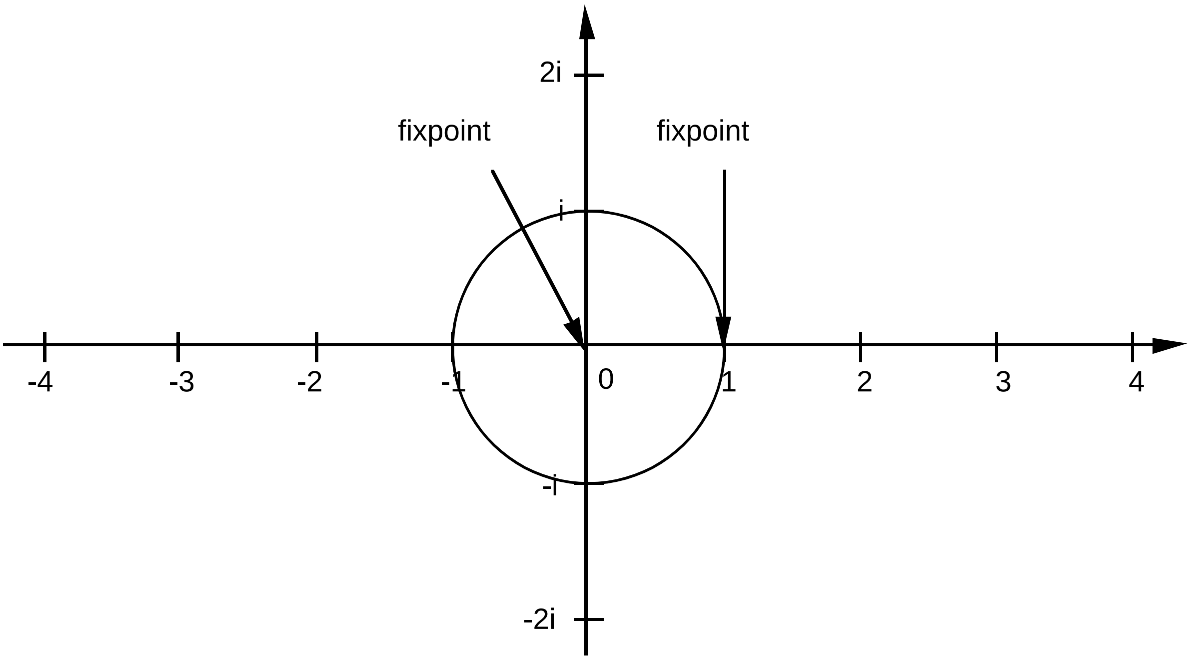
Iteration in the Complex Number Plane

Before reading this chapter, I strongly recomend the dear reader to reapeat the very first chapter *Iteration.* We will now peform the same dynamical process. However, instead of restricting ourself to the real number line, we will see what will happen if we peform the same iteration rule on the complex number plane. A point in the complex number plane we call *z*, where $z = z_{real} + iz_{imag}$, z_{real} being the real (horizontal) axis and z_{imag} being the imaginary (vertical) axis. Now, if we mark all those "z" which do NOT attracts to infinity for the dynamical process $z \to z^2$ we obtain a disc wth the radius 1 (see below).



With respect to the dynamical process $z \to z^2$ there are three kinds of points on the number plane:

1) Points outside the circle have orbits that are attracted to infinity.

2) Points inside the circle have orbits that are attracted to zero.

3) Points on (and then I mean EXACT on) the circle have orbits which never in infinities infinity will leave the circle. These numbers (= the unit circle) constitutes the Julia set for the dynamical process $z \to z^2$.

Points on the Julia set together with a domain that eventually is enclosed by the Julia set, in our case numbers that are situated on and inside the circle, we call the *filled-in Julia set.* The filled-in Julia set uses to be colored black, if no special inside coloring methods or filters are used.

Which orbit has a (starting-)number on the circle? As we seen in "iteration" our dynamical process has one repelling fixpoint z = 1. This one attracts itself (1 -> 1 -> 1 etc) and the preperiodic point z = -1 (-1 -> 1 ->1 etc). When we only took the real number line into account, these two points the whole Julia set. However with our enlarged view on the world (the previous chapter) we see these are only two points on a circle. Then there are a lot of more preperiodic points those orbits attract to the fixpoint z = 1, for example z = i (at the very top of the circle) having the orbit i -> -1 -> 1 ->1 etc (as $i^2 = -1$), or z = -i (at the very bottom of the circle) having the orbit -i -> -1 -> 1 ->1 (as $(-i)^2$ also equals -1). Although infinity many points are preperiodic to our fixpoint, those do not constitute the whole circle. There are also points, so called repelling peroidic points, that have periods of 2, 3, …… etc, for short all cycles of all periods. To those there are also preperiodic points. Finally there are points on the circle those orbit are complete chaotic, that is during iteration never in infinity reach the same spot anymore on the circle.

In other words, the circle that in all times being a symbol on divine order in fact is constituted by infinite chaos. In this chapter the following obscure words have benn introduced:

- *filled-in Julia set,*

- *repelling periodic point.*

Now my dear reader is fully introduced to read may very first article in my chaotic series, *Level sets.*

Regards
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