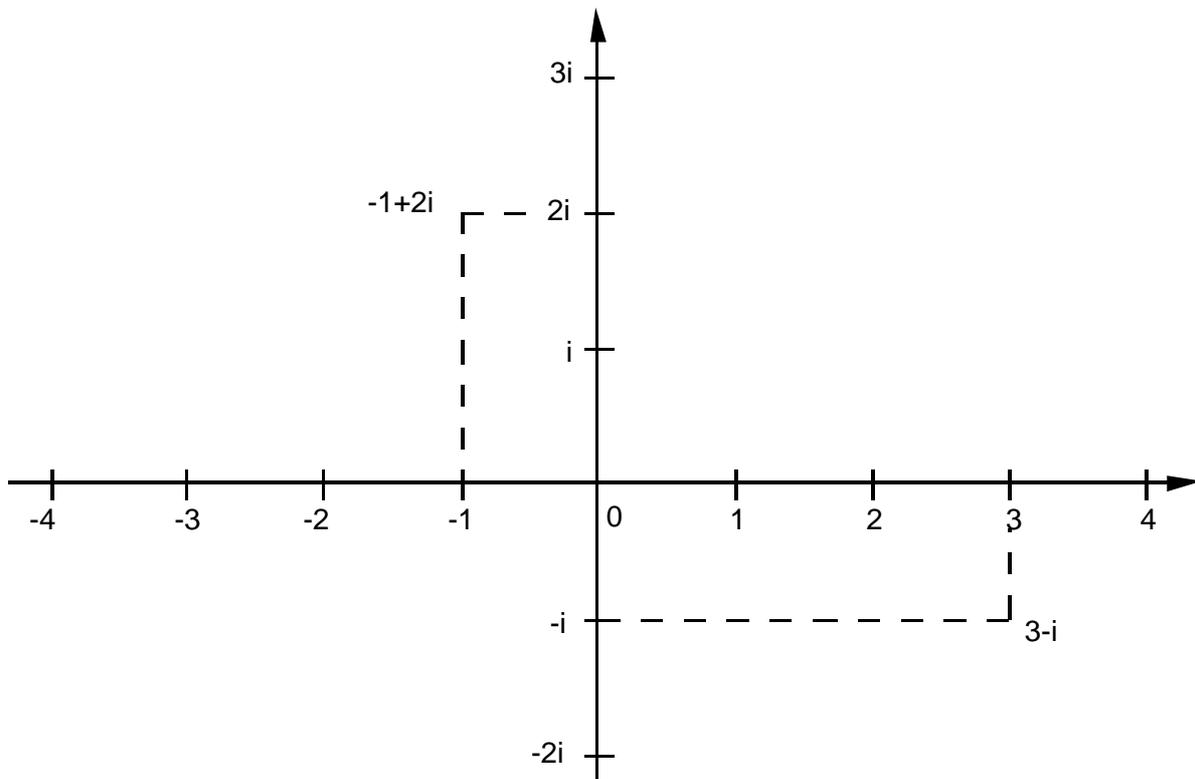


The Imaginary Numbers and the Complex Number Plane

In the first intro, *Iteration*, we were occupied by squaring numbers. The opposite is to draw the *square-root* of a number. The meaning is to search for a number which squared gives the given number. Example, $\sqrt{9} = \pm 3$ as $3^2 = 9$ and also $(-3)^2 = 9$, $\sqrt{16} = \pm 4$ as $4^2 = 16$ and also $(-4)^2 = 16$ etc. Most square-roots however are not rational numbers, for example $\sqrt{2} = \pm 1.41421..$ and $\sqrt{3} = \pm 1.73205... .$ Now what will happen if we draw the square root of a negative number, for example $\sqrt{-1}$? Not 1 because $1^2 = 1$, neither -1 because $(-1)^2 = 1$. In other words, there are no number which squared (multiplied with itself) becomes -1 (and no other negative numbers either). However there are no reasons why not do an invention of such number as long as they not are confused with the ordinary real numbers (rational numbers and not rational numbers)! This is exactly what the mathematicians have done. They simply declare that $\sqrt{-1} = \pm i$, "i" meaning *imaginary*. Thus $i^2 = -1$ and $(-i)^2 = -1$. Now it's possible to draw the square-root of other negative numbers also, for example $\sqrt{-9} = \pm 3i$, $\sqrt{-16} = \pm 4i$ etc.

These imaginary number we can put on a number line of it's own. For not confusing it with the ordinary real numbers we place it perpendicular to the real number-line end now we have a plane, the *complex number plane* (note complex means 'put together', not 'complicated'). A complex number thus is made up of two components, the *real part* and the *imaginary part* marked with "i". The illustration below shows how the complex numbers $-1+2i$ and $3-i$ are situated in the complex number plane.



To count with complex numbers is not much more difficult than counting with the ordinary real numbers. For example, let's add the two complex numbers above, $-1+2i$ and $3-i$. We simply add the real parts and imaginary parts separately, thus $(-1+2i) + (3-i) = 2+i$. In like manner when subtracting; $(-1+2i) - (3-i) = -1 + 2i - 3 + i = -4+3i$ (when deleting the brackets after the minus sign, we change the sign in that were inside the brackets. When multiplying we simply multiply each of the terms with each other, thus $(-1+2i)*(3-i) = -3 + i + 6i - 2i^2 = -3 + 7i - 2i^2$. As $i^2 = -1$ according to the definition of imaginary numbers, we obtain $-3 + 7i - 2(-1) = -3 + 7i + 2 = -1+7i$. Division there is no need to talk about here (one first prolongs with the conjugate quantity of the denominator).

Now I hope that the term "complex numbers" not sounds so mystical anymore. The important thing to know is that a complex number has two components, *a real part* and an *imaginary part*, and that it shows up on a plane, the *complex number plane* instead of a line. In fact the ordinary real numbers can be regarded as a special case where their imaginary parts equals zero. Mathematically speaking the set of the real numbers is a *subset* of the complex numbers, just as the set of rational numbers is a subset of the real numbers, etc.

 Regards
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