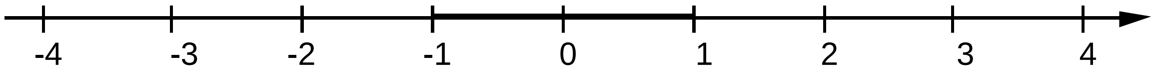
Iteration

Iteration means ’redoing’. What will happen if we take a number, square it, square the result again in infinitum, expressed in mathematics $x \to x^2$ (read x becomes x^2, ”x” is any number, the pile means that the result, i e the squared number, becomes the new starting number etc in infinity). In most of the cases the result will increase beyond any limit, for example $2 \to 2^2 = 4 \to 4^2 = 16 \to 16^2 = 256$...................-> +infinity. However a number less than 1, but bigger than -1 tends to zero, for example $1/2 \to (1/2)^2 = 1/4 \to (1/4)^2 = 1/16 \to (1/16)^2 = 1/256$...................-> 0. The series of numbers you will obtain for a certain starting number is called the *orbit* of that starting number. The orbit for the number 2 for the dynamical process $x \to x^2$ therefor is 2 -> 4 -> 16 etc. If we on the number line mark up every number NOT tending to infinity under iteration for the above dynamical process, we will obtain a straight line from -1 to +1 (see below).



As any negative number squared will produce a positive product, all starting values to the left of zero will result in the same orbit from the second iteration as corresponding positive starting values, for example $-2 \to (-2)^2 = 4 \to 4^2 = 16$ etc. Numbers outside the thick line will result in orbits tending to plus infinity, and numbers inside the limit points of the thick line have orbits tending to zero. In other words for the dynamical process $x \to x^2$ infinity is an *attractor* when the initial number is bigger than 1 or less than -1, and zero is an attractor for number between -1 and 1.

What will happen with the end points of the thick line when the above dynamical process is running? The end point to the right does not change at all as 1*1 = 1. Such a point is called a *fixpoint*. The end point to the left, -1, will give the orbit $(-1)^2 \to 1 \to 1$ etc, i e as for +1 after one iteration. Such a point is called a *preperiodic* number. That's because after a certain number of iterations, in our case one iteration, the orbit will reach its periodic cycle, in our case a 1-periodic cycle. The fixpoints to the process $x \to x^2$ we will obtain by solving the equation $x = x^2$. As it is a second-degree equation it ought to have 2 solutions. Our equation is so simple that can see directly that the solutions are $x = 0$ and $x = 1$, because $0 = 0^2$ and $1 = 1^2$. That means that also $x = 0$ is a fixpoint. The two fixpoints 0 and 1 however have quite different characters. Above we called $x = 0$ an attractor because all the numbers between -1 and 1 have orbits attracting to this point during iteration. Such a fix point is called an *attractive fixpoint,* or simply *attractor* (in this case the fixpoint is also *superattractive*, but that will not be described here). With the other fixpoint, 1, the case is quite different. It only attracts two points on the numberline, itself and the preperiodic number -1 and *exact* only these two numbers. Such a fixpoint is called a *repelling fixpoint,* or just *repeller.* Let’s for example take the numbers 0.999 and 1.001. Although they are situated very close to the fixpoint 1, their orbits will not come closer to this fixpoint, the first one

(0.999) will approach zero (the attractive fixpoint), and the other one (1.001) will approach infinity. In fact infinity is also an attractor as inf = inf^2 but when solving equations one uses to only take the finite solutions into account. In other words, a repelling fixpoint repels every point except itself and a number of preperiodic points.

These two points, -1 and 1, which constitute the border between those numbers whose orbits tends to infinity or attracts to zero, constitute the *chaotic set* for the dynamical process x -> x^2. That means that an infinite little ”miss step” on the side of these points (for example 0.999 or 1.001 instead of 1) will result in a quite different fate. Such a sensitivness for an exact start value is called *sensitive dependence on initial condition* (compare the butterfly who move its wings in the tropical forest in Burma and because of that causes an hurricane on the north Atlantic to take another orbit some months later). This chaotic set is also called the *Julia set* for the dynamical process x -> x^2. In order to come closer these Julia sets, which mostly are fractals, we first have to expand our minds from real number to *complex numbers.* However that will be the task for the next chapter of the intro. First, however we will list the obscure terms introduced in this chapter;

- *iteration,*
- *orbit,*
- *attractor,* which can be *attractive* or *repelling,*
- *preperiodic point,*
- *chaotic set,*
- *sensitive dependence on initial condition,*
- *Julia set.*

With these obscure terms we will reach far when chattering about fractals and chaos.

Regards
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